Effects on Risk Taking Resulting from
Limiting the Value at Risk or the Lower Partial Moment One

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Abstract

We demonstrate that limiting the value at risk may increase banks' risk taking while the expected return remains unchanged. Therefore, value at risk is neither a good internal nor a good regulatory risk measure. The lower partial moment one is shown to be a superior alternative.

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1 Introduction

The Capital Adequacy Directive (CAD) issued by the European Union (EU) and the „Basle Capital Accord to Cover Market Risk" (Basle Accord) issued by the Bank for International Settlements (BIS) limit banks' risk taking via a bound on the risk/equity ratio. Risk may either be calculated by in-house models (value-at-risk concepts) or by certain standard methods.

Normatively (if one accepts the axioms put forward by von Neumann/Morgenstern)[1], banks make investment decisions such as to maximize expected utility. This implies a certain notion of risk. If the notion of risk underlying the regulation differs from that used by a bank in its investment decisions, adverse effects on a bank's risk taking may result from regulation, i.e. the regulatory limit might in some cases cause banks to take a riskier position than they would without any limit.

The risk measure prescribed for banks in their in-house models is value at risk (VaR). In a companion paper[2], we have shown that VaR may deliver a risk ranking that is not compatible with expected utility maximization. In this paper, we will show the implications of this result for banks' risk taking.

The paper is organized as follows: In Section 2 we will lay the foundations for decision making under uncertainty. Section 3 introduces the basic example used throughout the paper. As a reference situation we examine the asset allocation without any regulation in Section 4. There, we mention mean preserving spreads as a key concept to rank distributions according to riskiness. Section 5 is then devoted to asset allocation when the VaR is limited. After having discussed some practical problems of the VaR concept in Section 6, we turn to the lower partial moment one (LPM₁) as an alternative to VaR (Section 7). Section 8 concludes by summing up main results and issues for further research.

2 Decision Making Under Uncertainty

Suppose a bank may choose between alternative risky portfolios which are represented by their probability distributions. The information contained in a probability distribution is often condensed into two characteristics: a return and a risk figure. In the one period case with equal initial investments, expected payoff and expected rate of return yield identical rankings. As risk measures, variance, standard deviation, VaR, and many other indices are possible. If the portfolios happen to have identical expected payoffs, the importance of the risk component of the distributions is isolated.

If we want to know which of the portfolios a bank[3] will choose in order to maximize its expected utility, we need to have some information on the bank's utility function. We assume that banks' utility functions are

- non-decreasing (i.e. banks prefer more wealth to less) and
- concave (i.e. banks are risk averse).

Choosing between two portfolios with the same expected payoff, a risk averse bank deciding according to return and risk will never pick the riskier portfolio. If maximizing expected utility, this statement only holds for certain notions of risk.

In our companion paper, we have argued that the literature knows three concepts of risk which are compatible with expected utility maximization for non-satiated, risk averse investors. A portfolio g is riskier than a portfolio f with the same expected payoff if

- f shows second-degree stochastic dominance over g[4],
- g has more weight in the tails than f, i.e. g can be obtained from f by a series of mean preserving spreads,
- g is equal to f plus noise.[5]

In the following, we will concentrate on the second of these equivalent concepts, the mean preserving spread, because it will prove to be quite intuitive for our purposes.

3 Basic Example

For ease of exposition, we will assume throughout the paper that a bank may choose, at t=0, between three alternative portfolios of securities: f, g and h. At t=1, the three portfolios will all have payoffs between -5 and +4, the set of possible payoffs being discrete. The three portfolios differ in the probabilities with which the possible payoffs occur (Table 1).

Payoff	Portfolio		
	f	g	h
-5	0.5%	1%	0.25%
-4	1%	0.5%	0%
-3	1.5%	1%	0%
-2	1.5%	2%	0%
-1	3%	3%	0%
0	22.5%	22.5%	43.6875%
3	40%	40%	0%
4	30%	30%	56.0625%
Expected Value	2.23	2.23	2.23

Table 1

4 Asset Allocation Without Regulation

4.1 Mean Preserving Spread

Let us compare the probability distributions of f and g to get an idea about the notion of risk every non-satiated, risk averse expected utility maximizer has. For the payoffs from -1 up to 4, the probabilites of f and g are identical. f and g only differ in the probabilities associated with the payoffs -5, -4, -3 and -2. If we look at the probabilities of these payoffs, we recognize that the probability function g can be

obtained from f by the following steps: We start from f and take probabilty weight of 0.5% from the payoff -4 and shift it to the payoff -5, that means shift it further to the left. Then we take probability weight of 0.5% from the payoff -3 and shift it to the payoff -2, i.e. we shift it further to the right. Figure 1 illustrates this.
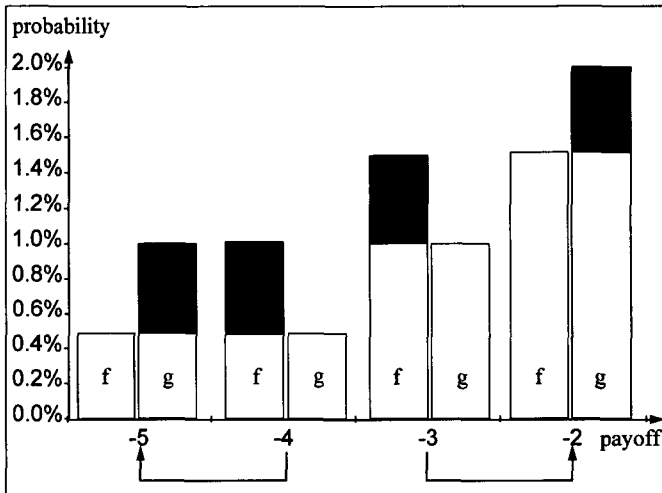

Figure 1: Mean Preserving Spread

We see that we can obtain g from f by taking away probability weight from two payoffs x and y (x < y) and adding it to two different payoffs, wherein the probability weight taken from x is shifted further to the left and the probability weight taken from y is shifted further to the right. The shift to the left-hand side decreases the mean, while the shift to the right-hand side increases the mean. The shifts are performed such that these two opposing effects compensate each other, i.e. the mean does not change. A shift of probability weight performed in this manner is called a mean preserving spread (MPS).[6]

Every non-satiated, risk averse investor maximizing expected utility would choose f over g[7] thereby implicitly agreeing (because the means are equal) that distribution g is riskier than distribution f. That is to say, if g can be obtained from f by either a single mean preserving spread or a series of mean preserving spreads, we know that no

rational individual would pick g over f. In other words, for every utility function from the class of increasing, concave utility functions, the expected utility of f is greater than the expected utility of g.

As noted above, if g can be obtained from f by a single MPS or a series of MPSs, then g is said to have more weight in the tails than f. Rothschild/Stiglitz use this expression to illustrate that in a mean preserving spread „some of the probability weight [is taken] from the center of f and [added] ... to each tail of f in such a way as to leave the mean unchanged".[8] This illustrates what is happening, though, strictly speaking, it is not quite correct. In our example, in shifting probability weight from the payoff -4 to -5, we did indeed shift probability weight from the center of f to the tail of f. But in shifting probability weight from the payoff -3 to -2, we actually shifted probability weight closer to the center of f. However, for simplicity and because the expression is widely known, we will nevertheless stick to saying that g has more weight in the tails than f.

It turns out that f and h do not differ just by MPSs: f has more weight on the lowest payoff (-5), and h has more weight on the highest payoff (4). Hence neither one can be obtained from the other by shifting more weight to the tails. Likewise, g and h cannot be ordered by MPSs.

In our example, only f and g can be ordered pairwise according to riskiness in terms of MPSs. Consequently, the concept of MPSs yields only a partial ordering. As a matter of fact, many distributions cannot be ordered by this concept.

4.2 The Efficient Set

As mentioned above, it depends on the bank's utility function which of the three portfolios will be chosen. It will most likely differ from bank to bank. Thus, we cannot make a general statement. However, due to the assumptions made, we do have some information that holds for all banks' utility functions[9]. (We have assumed that

they are increasing and concave.) As stated in the preceding section, the expected utility of f is higher than the expected utility of g for all conceivable increasing and concave utility functions. Thus, we know that no bank will choose g.

The efficient set is the set of portfolios that contains the optimal portfolios for *all* conceivable individuals with utility functions from the assumed class, and only those portfolios. Since g can be obtained from f by an MPS, g cannot be the optimal portfolio for anybody. Therefore, g is no member of the efficient set. The efficient set (given equal means) only includes portfolios which cannot be compared by MPSs, in our case portfolios f and h.

4.3 Optimal Portfolios

Let us look at the asset allocation decisions of some specific banks.

Bank A, for example, has the following (increasing and concave) utility function:

$$\text{Utility (payoff)} = \ln (\text{payoff} + 1,000) \cdot 1,000,000,000 - 6,909,980,000.$$

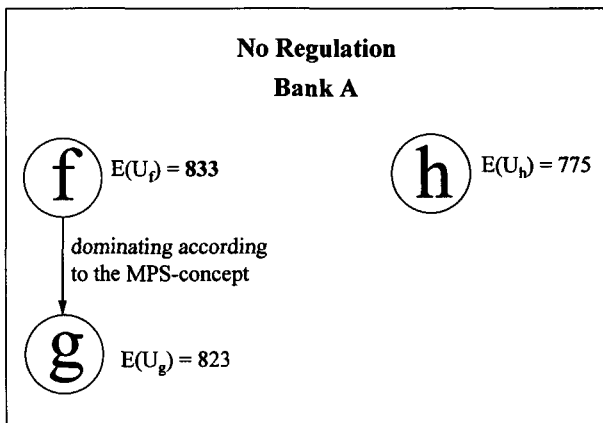The resulting values for the expected utilities E(U) of the three portfolios f, g and h are shown in Figure 2:

Figure 2: Asset Allocation of Bank A Without Regulation

Bank A maximizes its expected utility by investing in portfolio f.

Bank B, on the other hand, might have the following (increasing and concave) utility function:

$$\text{Utility (payoff)} = \ln(\text{payoff} + 10) \cdot 10,000 - 24,080.$$

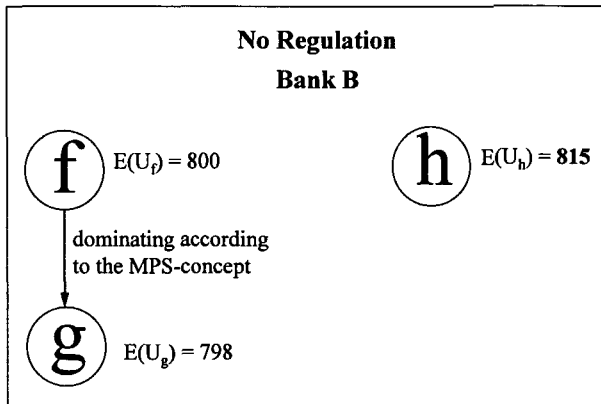The resulting values for the expected utilities E(U) of the three portfolios f, g and h are shown in Figure 3:

No Regulation
Bank B

$\left(\text{f}\right)$ E(U_f) = 800

dominating according
to the MPS-concept

$\left(\text{g}\right)$ E(U_g) = 798

$\left(\text{h}\right)$ E(U_h) = **815**

Figure 3: Asset Allocation of Bank B Without Regulation

Bank B maximizes its expected utility by investing in portfolio h.

Within the given class of utility functions, we have found a bank that chooses portfolio f and a bank that chooses portfolio h. However, it is not possible to construct a utility function belonging to this class that would lead a bank to choose portfolio g.

In the following, we will have a closer look at Bank A, the bank that chooses portfolio f in the absence of regulation. We will ask how Bank A's asset allocation decision changes in the presence of regulation.

5 Asset Allocation Under Regulation
5.1 The Regulatory Authority's View of Risk

We have argued that the risk ranking derived from the concept of mean preserving spreads is the correct risk assessment for all non-satiated, risk averse investors, because of its compatibility with expected utility maximization. However, since the regulatory authority does not take the position of an investor, the MPS risk assessment is not necessarily the correct one from the regulatory point of view. Having objectives like protecting depositors and preventing bank runs and systemic crises in the banking industry[10], the regulatory authority might very well have a view of risk that is distinctively different from that of an investor.

In order to be able to derive a concept that gives an appropriate risk assessment from the regulatory point of view, we would first need to know something like the regulatory authority's objective function. As far as we know, a formal, operational objective function for the authority that regulates banks has not yet been derived in the literature. We, too, will not offer a suggestion for such an objective function. Instead, we will argue based on an idea which is similar to the concept of efficient sets; i.e. we will argue based on the idea that certain portfolios will (normatively) never be prefered over others by the regulatory authority.

Consider two portfolios, f and g, where g can be obtained from f by one or more mean preserving spreads. Portfolio g then has more weight in the tails than f. It seems highly plausible that a regulatory authority would always prefer a bank to invest in the portfolio that has less weight in the tails, especially because it would prefer a bank to have as little weight as possible in the left tail (i.e. on the losses). Thus, without any further knowledge of the regulatory authority's objective function, it seems safe to say that any regulatory authority would want a bank to invest in f rather than in g.

We cannot make a statement whether the regulators should prefer f or h. At first glance it seems plausible that h is prefered to f by regulators because the negative payoffs have always lower probabilities for h than for f. But, without the objective function of the regulators, we can only say that they should not want the banks to choose g, an investment that results from f by a mean preserving spread.

We have seen that in the absence of regulation no (non-satiated, risk averse) bank would ever choose g over f if g can be obtained from f by one or more mean preserving spreads. Regulation of banks' risk taking should not change this.

5.2 Value at Risk

VaR is the concept prescribed by the Basle Accord for banks' in-house models. In-house models may be used instead of the standard methods to calculate the amount of equity that a bank is required to have to cover the risk contained in its trading book.

VaR is defined as the loss that with a certain (one-sided) confidence level will not be exceeded over a certain time period. In other words, the VaR of a portfolio is the loss that will be exceeded only with a certain probability (1 minus the confidence level) according to the given probability function.

VaR is a loss figure, where the loss is defined as the initial investment amount minus the payoff at the end of the period.[11] For simplicity, we assume the initial investment amount to be zero. The loss then equals the negative of the payoff at the end of the period. Thus, if the payoff is negative, the loss and therefore the VaR is positive. The higher the (positive) VaR, the riskier the investment according to this concept.

In our example, for simplicity the (maximum) probability of exceeding the reported VaR is set to 2.75% by the regulatory authority, i.e. the prescribed confidence level is 97.25%. (In reality, it is usually set to 99%.)

Given our confidence level, we can calculate the VaRs[12] for the portfolios f, g and h. We demonstrate the method with portfolio f. For portfolio f, the payoff -5 is the lowest payoff possible; it cannot be exceeded. The second lowest possible payoff, which is -4, can only be exceeded by the payoff -5 with a probability of 0.5%. Next, we look at the payoff -3. This payoff can be exceeded by the payoff -5 with a probability of 0.5% and by the payoff -4 with a probability of 1%, i.e. with a total probability of 1.5%. This probability is still smaller than the maximum probability allowed by regulation (2.75%). As the probability for the payoff -3 is 1.5%, the payoff -2 would be exceeded with a probability of 3%, which is more than permitted. The highest payoff that will not be underscored with a probability of no more than 2.75% thus is -3. The VaR for f then is 3. Calculating the VaRs for the other distributions analogously, we get a VaR of 2 for g and a VaR of 0 for h.

This means, according to the Basle Accord's notion of risk, we have the following risk ranking: f is riskier than g, and g is riskier than h. Compare this to the risk ranking derived from the concept of mean preserving spreads: there we found f to be less risky than g, which is the opposite of the VaR's assessment. Moreover, as VaR delivers a complete ordering, h can be risk-ranked in comparison to f and g, whereas with the concept of mean preserving spreads we could neither compare f and h nor g and h.

5.3 Effects of Value at Risk on Risk Taking

In order to see whether regulation with the concept of VaR might indeed generate the adverse effect of leading a bank to choose g instead of f, we will look at a bank which chooses f in the absence of regulation: Bank A in our example.

The asset allocation decision of Bank A under regulation is limited by the amount of equity of Bank A. Let us assume that its supply of equity allows Bank A to take on a maximum VaR of 2.4.

In a first step, let us assume that portfolio h does not exist. That is, Bank A may principally choose between portfolios f and g. Since the VaR of f (which is 3) is higher than permissible, whereas the VaR of g (which is 2) is low enough to be permissible, Bank A must choose portfolio g under regulation. Bank A will rather invest in g than not invest at all because investing in g delivers a higher expected utility (823) than not investing and thus receiving a payoff of 0 at t=1 (which delivers an expected utility of -2,224,721).

In the absence of regulation (and of h), the efficient set consisted exclusively of portfolio f. If under regulation portfolio f is no longer permissible for Bank A, because its VaR is too high, the complete efficient set disappears for Bank A. Bank A is thus forced by regulation to choose a formerly inefficient portfolio (cf. Figure 4).
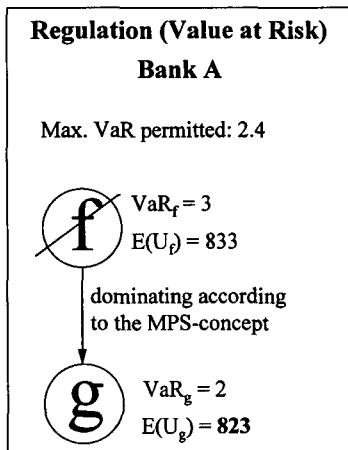

Figure 4: Asset Allocation of A (VaR limited; h not available)

Thus, regulation based on the concept of VaR may actually alter a bank's asset allocation decision in a way not intended by the regulation: The portfolio Bank A chooses under regulation is riskier than the portfolio Bank A would choose in the absence of regulation. In this case regulation, which intends to limit risk taking, leads to higher risk taking. As we have argued earlier, if a bank's investment decision

changes due to regulation from portfolio f to portfolio g, which is dominated by f according to the MPS concept, every regulatory authority should agree that this regulation is inappropriate.

As we have seen, if Bank A may only choose between portfolios f and g, regulation erases the complete efficient set. The question then is whether the complete erasion of the efficient set is necessary for the possibility of banks choosing inefficient and more risky portfolios under regulation.

To answer this question, we take portfolio h back into the set of portfolios available. The efficient set then consists of portfolios f and h. Portfolio f, again, is eliminated by regulation. However, the other element of the efficient set, portfolio h, is permissible under regulation, because its VaR of 0 lies below the maximum VaR allowed for Bank A.

Intuitively, one would suspect that Bank A will now choose the remaining element of the efficient set, portfolio h. However, comparing the expected utilities Bank A obtains from g and h, we see that Bank A's expected utility from investing in g is higher than that from investing in h. This means that Bank A will again choose g under regulation (cf. Figure 5).
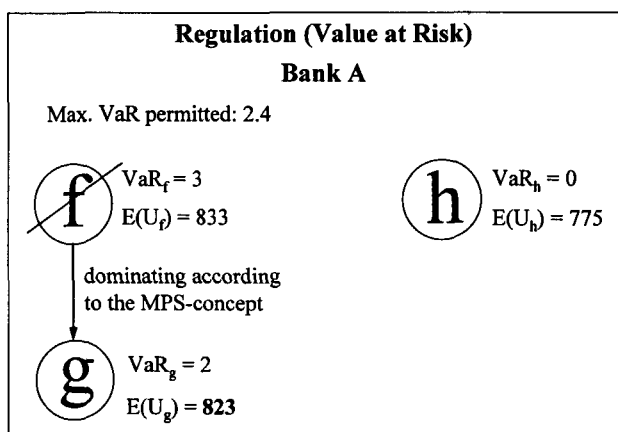

Figure 5: Asset Allocation of Bank A (VaR limited)

Thus, even if elements of the efficient set remain permissible under regulation, it may happen that a portfolio with a high VaR is excluded by regulation though it is dominating another portfolio according to the concept of mean preserving spreads. If that dominated portfolio has a low VaR and is permissible and if it is not dominated by another permissible portfolio, the dominated portfolio is contained in the efficient set under regulation. Consequently, a bank may choose this dominated portfolio and take more risk than without regulation.

Of course, there might be banks (e.g. Bank B) that prefer, on account of their utility function, h to f. In this case, we would not have the adverse effect on risk taking just demonstrated for our example. Thus, we do not claim that regulation *always* results in higher risk taking of banks, but that indeed it *might*. This conceptual problem of the VaR[13] should be a source of concern for regulators.

6 Practical Problems of the Value-at-Risk Concept
6.1 Misspecified Distributions and Standard Methods

We have seen that the use of VaR by the regulatory authority may result in the efficient set changing in such a way that some banks might actually take more risk under regulation than they would without. In showing this effect, we have assumed that the portfolios' payoff distributions are exactly known. In most instances, however, this is not going to be the case. Sampling errors, erroneous distribution assumptions, and deliberate gaming may lead to a reported VaR which is not a correct representation of the portfolio's actual payoff distribution. This will have additional effects on the composition of the efficient set, which in turn may worsen the problem described above.[14]

The same holds for the use of standard methods. Because of the standardization, they will generally misrepresent a portfolio's true VaR. This, again, might have adverse effects on the efficient set.

6.2 The Variance-Covariance Method and Asset Allocation

In their in-house models, banks usually use one of the following three methods for computing the VaR: the variance-covariance method, historical simulation or Monte Carlo simulation (stochastic simulation).[15]

The variance-covariance method is based on the assumption of normally distributed portfolio returns. For a normal distribution and any confidence level, the VaR can be computed from just two moments of the distribution: the arithmetic mean (μ) and the standard deviation (σ). In our example, the (one-sided) confidence level is set to 97.25%. This corresponds to a z-value (standard normal distribution) of -1.919. The VaR with a one-sided confidence level of 97.25% for any normal distribution is then calculated as[16]:

$$VaR(2.75\%) = \mu - (-1.919) * \sigma.$$

Since our three portfolios have the same mean, the ranking of their VaRs according to the variance-covariance method is determined by the ranking of their standard deviations and thus their variances.

Calculating the VaR with the variance-covariance method for our portfolios (erroneously assuming normal distributions, but using the correct means and standard deviations) we get a VaR of 1.5777 for f, a VaR of 1.5874 for g and a VaR of 1.6365 for h. If we still assume that its equity allows Bank A to take on a maximum VaR of 2.4 then all three portfolios are permissible when calculating the VaR with the variance-covariance method.

This shows a major problem of calculating the VaR with the variance-covariance method: whenever the payoffs are not normally distributed the VaR computed with the variance-covariance method is wrong and therefore it does not indicate the true loss which is only exceeded with a certain probability. In our example the probability of exceeding the VaR, i.e. observing outcomes below the negative of the values given

above, is 4,5% for f, 4,5% for g and 0,25% for h. Consequently, the probability level set by the regulators is no longer kept if the VaRs are not computed exactly. Thus, there might be some portfolios that are permissible according to the wrongly computed VaR while they are actually too risky from the regulator's point of view.

As the VaRs calculated with the variance-covariance method do not restrict the investment decision of Bank A, they cannot produce the adverse effect on risk taking we found with the exactly computed VaR in the preceding section. Bank A can and will invest in f, the portfolio with the highest expected utility (cf. Figure 6). The question to be answered is whether these adverse effects of risk taking are always avoided if the VaR is calculated on the basis of the variance-covariance method.
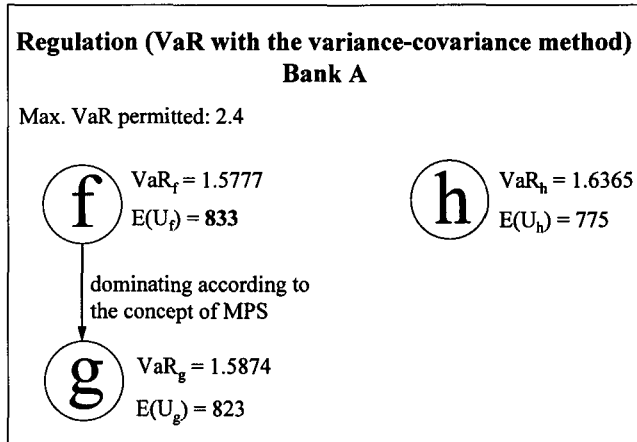

**Regulation (VaR with the variance-covariance method)
Bank A**

Max. VaR permitted: 2.4

f — $VaR_f = 1.5777$, $E(U_f) = 833$

h — $VaR_h = 1.6365$, $E(U_h) = 775$

dominating according to the concept of MPS

g — $VaR_g = 1.5874$, $E(U_g) = 823$

Figure 6: Asset Allocation of Bank A (VaR according to the Variance-Covariance Method)

Now, suppose for a moment that h is not permissible because the regulator accepts only a maximum VaR of 1.6 for Bank A. The efficient set under regulation then only includes f. We know that whenever g is more risky than f according to the concept of mean preserving spreads, the variance of g is higher than that of f.[17] (This requires distributions with equal means.) That is to say, the risk ranking delivered by the value-at-risk concept based on the variance-covariance method is compatible with

expected utility maximization in all cases where the portfolios considered can be compared by the concept of mean preserving spreads. Thus it can never happen that f is excluded from the efficient set while it is still permissible to invest in g. To make it perfectly clear: Calculating VaRs with the variance-covariance method avoids that the risk ranking according to the VaR contradicts the risk ranking according to the concept of mean preserving spreads. This is true even if VaR provides wrong values for the loss exceeded only with the maximum probability set by regulators.

7 Lower Partial Moment One and Asset Allocation

7.1 Motivation and Definition of Lower Partial Moment One as a Risk Measure

Up to now we have considered a regulatory risk measure, VaR, that does not care about the extent of losses in case of bankruptcy. This tacitly assumes that the amount depositors lose if there is a collapse does not matter for the confidence in the banking system and the danger of a bank run.

If there exists a deposit insurance, then depositors should indeed get back their money from this institution. But as the deposit insurance's ability to pay usually is not infinite, it may happen that the depositors are not paid all their deposits if simultaneously several banks are hit by disadvantageous market trends. So even in a world where deposit insurances exist, the amount of losses does not seem to be unimportant.

If one looked at the area below the cumulative density function up to a given target payoff, this would be a risk measure which would consider not only the probability, but also the amount of losses. This measure is called Lower Partial Moment One (LPM_1).

The formal definition of the lower partial moment of order one with target t, $LPM_1(t)$, is

$$LPM_1(t) = \int_c^t (t - x) \, f(x) \, dx \, .^{18}$$

For all payoffs above the target, the target is reached and therefore the shortfall is zero: payoffs that are higher than the target cannot compensate payoffs below the target. Then, LPM_1 gives the expected amount by which the target is missed (the expected shortfall).

7.2 Using the Permitted Value at Risk as the Target

We have assumed in our example (Section 5.3) that the regulators do not want Bank A to exceed a loss of 2.4. Therefore we may take 2.4 as the target payoff t and then look, for any portfolio, only at those payoffs (with their corresponding probabilities) below t. Essentially, we only consider that part of the density function that lies to the left of t.

Calculating $LPM_1(2.4)$ for our three portfolios, we get the following values: 0.038 for f, 0.04 for g and 0.0065 for h. According to the LPM_1 concept, we therefore have the following complete risk ranking: f is riskier than h, and g is even riskier than f. Compare this with the risk ranking derived from the concept of mean preserving spreads: there we also found g to be riskier than f, but neither one could be compared to h. It is quite clear that we cannot say whether the risk ranking according to LPM_1 is reasonable concerning h, but concerning f and g it delivers the risk ranking the regulatory authority should indeed strive for.

7.3 Target Dependance of Lower Partial Moment One

We can easily demonstrate that the risk ranking of f and g depends on the chosen target. Just look at Figure 7 where the cumulative distribution functions of f and g are shown up to the payoff of -2. (The cumulative distributions for higher payoffs are

identical.) You can see that for any target smaller than -2 the LPM_1 of g is higher than that of f. That means for those targets LPM_1 delivers the same risk ranking as the concept of mean preserving spreads. For those targets that are equal to or higher than -2 the LPM_1s of g and f are identical. In these cases, LPM_1 does not recognize the difference in the distributions due to the mean preserving spread.
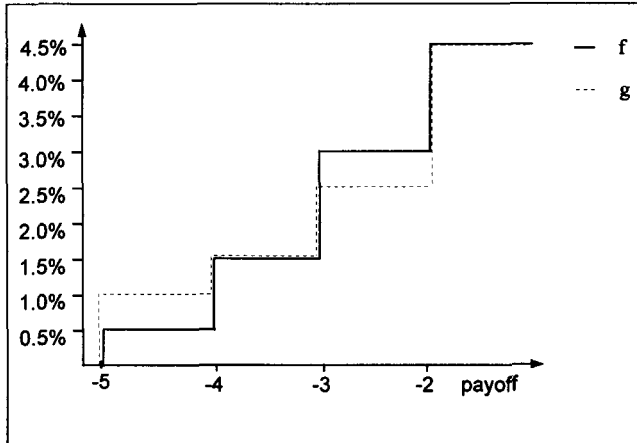

Figure 7: Cumulative Distribution Functions of f and g

The latter is not too serious if LPM_1 is used only as a regulatory restriction, because it does not exclude the less risky portfolio from the efficient set and therefore the bank is not forced to higher risk taking by regulation: For targets equal to or higher than -2, either g and f are both permissible or none of them is. (It can indeed be shown[19] that the risk ranking according to LPM_1 - unlike that according to VaR - cannot contradict MPSs' risk assessment, except when LPM_1 is equal for the two distributions to be compared.)

But the problem is serious for investors because they might not maximize their expected utility choosing randomly between portfolios with equal means and LPM_1s instead of looking for mean preserving spreads.

7.4 Limiting Lower Partial Moment One

Whenever regulatory authorities want banks to avoid investments that are too risky in their view, the regulators need an idea of when an investment is too risky. In the case of the LPM_1 concept, they have to fix (apart from the target) the maximum LPM_1 a bank may have. That maximum level of LPM_1-risk is always more or less arbitrary. However, this is not a disadvantage of the LPM_1 concept in contrast to the VaR approach, since the confidence level of the VaR concept is likewise arbitrary.

As we do not know normatively which LPM_1 a bank should be allowed from the regulator's point of view, we cannot say whether Bank A can invest in the portfolio f, the investment with the highest expected utility, or is forced to invest in h (cf. Figure 8). But whenever it is permissible to invest in one of our three portfolios, the inefficient portfolio g ist always excluded before the efficient portfolio f.
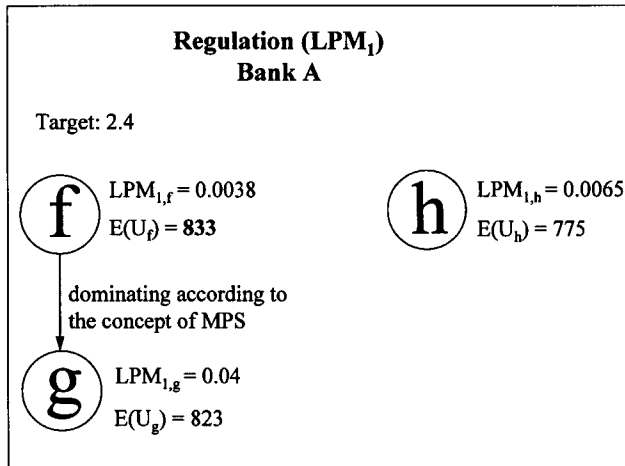

Figure 8: Asset Allocation (LPM_1 limited)

8 Conclusion

We may sum up the main results of our paper, some of which have formally been known from other branches of the economic literature.

Most of our analysis is based on the opinion that mean preserving spreads (MPSs) generate more risky distributions. Given this (debatable) judgement, we have shown, among others, that

- limiting the value at risk (VaR) may increase banks' risk taking,
- limiting the VaR may erase dominating distributions from the efficient set and add dominated distributions to it,
- VaR, therefore, is neither appropriate as an internal nor as a regulatory risk measure,
- wrongly assuming normal distributions when calculating the VaR via the variance-covariance method avoids risk rankings according to VaR that contradict those based on mean preserving spreads.

The lower partial moment one (LPM_1) was suggested as an alternative risk measure:

- Since it is compatible with expected utility maximization, no conflicts with MPSs can arise.
- However, equal means and equal values of LPM_1 for one target do not exclude the possibility of unique risk rankings according to riskiness in terms of MPS. In this sense, LPM_1 is target-dependent.

There are a number of issues for future research. Most importantly, an explicit objective function of the regulator (or at least a class of such functions) would be helpful to narrow down the number of reasonable risk measures.

Endnotes

1 Cf. von Neumann/Morgenstern (1953).
2 Cf. Guthoff/Pfingsten/Wolf (1997).
3 We abstract from any agency issues. For principal-agent conflicts compare Spremann (1987).
4 For an extensive bibliography of the literature on stochastic dominance see Bawa (1982) and Levy (1992).
5 For the second and third concept listed, cf. Rothschild/Stiglitz (1970).
6 For a formal definition of a mean preserving spread see Rothschild/Stiglitz (1970), p.227-229.
7 Cf. Rothschild/Stiglitz (1970), p. 237-240.
8 Rothschild/Stiglitz (1970), p. 226.
9 More about the utility functions of risk averse investors in Copeland/Weston (1992); p. 85-92.
10 Cf. Rudolph/Burghoff (1996), p. 17-29.
11 Cf. Albrecht/Bährle/König (1996), p. 5-6.
12 The VaR approach and the LPM_1 approach, which is presented later, are measures of shortfall risk and do not make assumptions about the return distribution, cf. Schröder (1996), p. 161.
13 For the contradictions and correspondences of VaR and the banks' notion of risk in more detail see Guthoff/Pfingsten/Wolf (1997).
14 Beder (1995) demonstrates the dependence of the Value at Risk on parameters, data, assumptions and methodology.
15 Cf. Beder (1995), JPMorgan (1995) and Linsmeier/Pearson (1996).
16 As the VaR is usually calculated for very short periods, μ is often set to zero in practical calculations. Compare for example JPMorgan (1995), p. 27. In this case, only the variances and covariances are calculated (or received from a supplier).
17 Cf. Rothschild/Stiglitz (1970), p. 241-242.
18 Cf. Fishburn (1977) p. 116.
19 Cf. Guthoff/Pfingsten/Wolf (1997).

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