

**SOLVENCY INSURANCE WITH OPTIONED PORTFOLIOS:  
AN EMPIRICAL INVESTIGATION**

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**Abstract**

In the recent literature, the return- and downside risk characteristics of portfolios with financial derivatives have been analyzed both in theory and empirically, using an asset-only framework. In this paper we will investigate the benefits of optioned portfolios for liability organisations such as pension funds and insurance companies, whose primary concern is the downside risk of the funding level, rather than the market exposure of the investments. Using the real-life data of a pension fund we will empirically show that the use of financial derivatives can create significant cost-neutral improvements of the solvency risk exposure of the pension fund, particularly if the optioned portfolios are constructed in view of (long term) prognoses of the development of the liabilities.

**Résumé**

Dans la littérature récente, le rendement et le risque de déficit de portefeuilles qui contiennent des produits dérivés ont été analysés aussi bien en théorie qu'en réalité. Cet article examine les gains de portefeuilles d'options pour des organisations comme les fonds de retraite et des sociétés d'assurances, qui s'intéressent surtout au risque de déficit du niveau de financement, plutôt qu'aux risques d'investissements. Nous utilisons les données d'un fonds de retraite réel pour montrer que l'emploi des produits dérivés améliore le risque d'insolvabilité, en particulier si l'on utilise des prédictions de développements d'obligations.

Key words: Financial Derivatives, Insolvency Risk, Asset / Liability Management

## 1 Introduction

**Integrated Asset / Liability Management (ALM)**, rather than asset-only portfolio management, has become of worldwide interest (cf. the recent volume “World Wide Asset and Liability Modelling” [Ziemba and Mulvey, 1997], and the references cited there). In the Netherlands ALM was initiated by pension funds in the nineteen-eighties, mainly to investigate the risks and benefits of structural changes of the investments from fixed income securities to equities and real-estate. Partly due to the resulting landslide in the asset allocations of the several hundreds of billions US\$ of the organisations (and the consequences for the funding), which resulted in benchmark portfolios of 60% fixed income and 40% risky securities, rather than the 90/10 ratio in the nineteen-eighties, other Dutch organisations have become active in ALM as well. This not only pertains to (internationally operating) insurance companies and banks, but also to other liability organisations such as building corporations and even (wealthy) individuals.

Shortly put, Asset / Liability Management, which is also referred to as Asset / Liability Modelling, can be described from two points of view (cf. [Boender, 1997a; Boender et al., 1997b]):

### 1. Management:

ALM concerns decision making about (dynamic) integrated strategies of the assets- and liability policy instruments of liability organisations, such that solvency constraints in terms of the funding level (= value of the assets / value of the liabilities) are satisfied at minimal cost.

### 2. Modelling:

ALM concerns the dynamic corporate modelling of the asset- and liability side of liability organisations, and the econometric modelling of the future environment of the relevant economic return and risk drivers of these organisations, such that either simulation procedures, optimization methods, or a mixture of both, can be employed to analyse, evaluate and support asset / liability decision making.

Of particular interest for these organisations is the focus on **solvency requirements**, which are formulated in terms of violations of legally required **funding levels**. That is, these organisations do not (any more) feel primarily exposed to the market risk of the investments, but to the impact of their decisions on funding level risk. In line with the downside risk concept in portfolio management (see also Section 2), our clients frequently measure these risks as the downside deviation of the **funding level**, which is defined as:

$$D = \left[ \int_{-\infty, 0} \varphi^2 f(\varphi) d\varphi \right]^{1/2},$$

where  $\varphi$  denotes the funding level (minus one), and  $f(\varphi)$  denotes the density function of  $\varphi$ .

Until recently, the policy instrument (= decision variables) to control the solvency risks was confined to the allocation of the investments to the basic asset classes cash, fixed income, equities and real-estate, which are determined in an integrated fashion with the funding and indexation agreements. In this paper we will investigate if efficiency gains can be accomplished by a dynamic use of **financial derivatives**. That is, we will

investigate for a real-life pension fund if “liability driven” strategies which actively employ options can create an efficiency gain in the contributions to be provided by the sponsor, and the funding risk exposure of the pension fund.

The outline of the paper is as follows:

- Since the benefits of derivative instruments heavily depend on the applied risk measures, Section 2 contains a brief discussion on the theory and practice of portfolio selection criteria.
- In Section 3 we develop a framework for dynamic option strategies, including strategies which particularly pertain to liability organisations.
- Section 4 describes our experimental design, which concerns a real-life defined benefit pension fund on the one hand and the modelling of the economic environment on the other.
- Section 5 presents and analyses the results of the efficiency gains in contribution rate and insolvency risk for different approaches to incorporate options in the investment policy. *Of particular interest will turn out the dynamic option policies which take into account the expected development of the funding level, especially if these policies are based on long term prognoses.*
- Section 6 concludes. Our experiments verify that the application of dynamic option policies to defined benefit pension plans yields significant efficiency gains with respect to the contribution rate and the downside deviation of the funding level. It also appears that the efficiency gain diminishes drastically if the probability of underfunding (which does not take into account the extent of underfunding) rather than the downside deviation is applied, but that the efficiency gains (as expected) further improve by the application of long term derivatives depending on long term prognoses of the liabilities. It is also verified that the efficiency gains (obviously) depend on the volatilities which are used to determine the Black-Scholes prices, and on the risk-characteristics of the pension plan, such that much theoretical and practical research remains to done.

## 2 Portfolio Selection Criteria

In the literature concerning appropriate portfolio selection criteria, there appears to be a marked contrast between the theoretical measures and those that are usually applied in the institutional investment industry. These differences are of special importance if derivative instruments are included in a portfolio, since the traditional mean-variance measures suffer from both theoretical and practical limitations when non-symmetrical asset return distributions are being used (see [Dybvig and Ingersoll, 1982; Bookstaber and Clarke, 1985] for a discussion on the integration of options in a mean-variance asset pricing model). If the standard deviation of return is used as a measure of risk, options often appear to be inefficient. As an example, consider the case of an investment in a stock index combined with a put option, to provide some downside protection. The perfect comovement of the option pay off and the index can't be captured by a variance measure, as it can not distinguish between outcomes where the put option ends up in the money and those where the put ends up worthless.

The **theoretical literature** in financial economics has not yet provided a widely accepted alternative measure for risk. Lower partial moments (LPM) were introduced as generalised risk measures in [Fishburn, 1977] (see also [Sortino and Van der Meer, 1990]), who showed the compatibility of this framework with the prevailing expected utility paradigm. In [Bawa and Lindenberg, 1977]) lower partial moments are embodied in an asset pricing framework, which generalised the traditional Sharpe/Lintner theory (see also [Harlow and Rao, 1989] for a generalization of the Bawa/Lindenberg approach). Moreover, LPM measures offer the possibility to calculate risk relative to a given benchmark return, which may be either deterministic or stochastic. Nevertheless LPM-measures appear not yet to have been fully integrated in the theory of finance.

Although the theoretical foundations of downside risk measures seem not yet completed, in **applications** lower partial moments have become a standard tool to analyse risk. See for example [Leibowitz, Kogelman and Bader, 1994] for the use of LPM measures to analyse portfolios with the primary equity- and fixed income asset classes. In [Adam, Albrecht and Maurer, 1996] these downside risk measures are used to investigate the risk and return characteristics of roll-over option strategies in an asset-only context, using monthly data for the 1960-1995 period of the German market. [Figlewski, Chidambaram and Kaplan, 1993] study the distributional characteristics of similar path-dependent roll-over option strategies.

In this applied paper we study LPM-measures of optioned portfolios for liability organisations, rather than in an asset-only framework. Two new risk measures will be considered, i.e. the probability of underfunding (= Zero-Partial Moment for liability organisations), but also the downside deviation of the funding level (= Second Partial Moment), defined in [Boender, 1997a and Boender et al, 1997b].

### 3 Put Option Strategies

Options can be included in a pension fund portfolio in many different ways. In this section, we will discuss different alternatives using index options, assuming a fixed benchmark asset allocation of stocks (i.e. the underlying index) and bonds:

#### I. Separate asset class in an asset-only framework:

In this alternative options are considered as a separate asset class, just as the primary asset classes of stocks and bonds. The amount to be invested in options is determined by applying either a simulation study or an optimisation procedure, both for the static and the dynamic case (i.e. path dependent varying the amount over time). However, in this approach the allocation decision is taken independently of any pension fund characteristics such as the development of the funding level.

#### II. Asset class to insure downside funding risk:

As an alternative, a path dependent option strategy is chosen whose characteristics are determined to provide an efficient insurance against downside funding risk. Clearly this approach necessitates a decision rule, which can either be based on expert opinion or on an (dynamic) optimization procedure, which prescribes which optioned portfolio should be chosen in view of the development of the key-drivers which determine the development of the funding ratio.

### III. Combined asset class:

Rather than considering financial derivatives as a separate asset class, a new asset class can be composed, consisting (of a portfolio) of options combined with an investment in the underlying index. Analogously to the approaches above, the amount to be invested in this combined asset class may be determined either by simulation or optimisation, enabling path-dependent (asset-only) roll-over strategies as well. As an example of this approach we refer to [Dert and Oldenkamp, 1996], who investigate a guaranteed return option portfolio which satisfies prespecified downside portfolio risk constraints.

### IV. Arbitrary return distribution:

In this approach options are used to construct the distribution of the pay-off function of the investment portfolio which optimally meets the risk profile of a (liability) organisation. To keep the required computations tractable, we could assume the derivative to be solely dependent on the stock index, where the dependence may be represented by some (nonlinear) function. This approach is similar to [Brennan and Solanski, 1981], where optimal insurance contracts are derived, considering only a single underlying variable. Note that the outcome of such a procedure may not correspond directly to an implementable investment strategy, as the optimal derivatives pay off function may be very irregular.

Evidently, the fourth approach is the most preferable from a theoretical point of view. The result of such a procedure consists of a pay off distribution of the optimal derivative instrument, which may be a complex function of the return on the underlying index. In theory, we may assume that such a contract can be acquired Over-The-Counter. However, since we primarily focus on the use of standard options in this paper, the optimal derivative according to this approach may not correspond to an implementable investment strategy. Therefore, we focus on the (path dependent) approaches I and II.

More specifically, the liability driven approach II is implemented as follows. Throughout the entire investment period (which equals 15 years), the asset allocation to the primary asset classes remains fixed, for instance a 40% stocks and 60 % bonds. Then, given:

- The asset allocation to the primary asset classes;
- The current funding level;
- The expected development of payments, contributions and liabilities;
- The expected development of the interest rates;

in each year the minimum return of the equities portfolio (including options) required to guarantee a pre-specified future funding level is computed. Subsequently, a time to maturity and exercise price for the options is determined such that these minimum return requirements are satisfied.

We further note with respect to our approach:

- In the experiments the approach is carried out where at the beginning of any year  $t$  the optioned portfolio is constructed to insure the funding level at the end of the year  $t$ , but we investigated also the approach to insure the funding level at the end of the year  $t+5$ .
- The option prices are calculated using the Black-Scholes formula for pricing European options. The volatility parameter is set equal to the historical volatility of the time series of stock prices that we used to estimate the model parameters (cf. Section 4).
- The premia to be paid for the options are borrowed against the short term interest rates (cf. Section 4).
- We take into account a maximum put premium (say 3% or 5% of the value of the equity portfolio).
- Since the construction of the optioned portfolio depends on estimates about the interest rates and the liability parameters, and since the required put premium may exceed preset limits, approach II obviously does not provide an absolute guarantee that the funding level does not violate the minimally required values. Whether these liability driven path dependent optioned portfolios nevertheless yield cost-neutral reductions of the funding risk is the subject of Section 5.

## 4 Experimental Environment

### 4.1 The Studied Pensionfund

The benefits of financial derivatives are analysed for a pension fund with the following characteristics:

- A defined benefit final pay pension scheme, which (still) is the prevailing pension scheme in the Netherlands.
- Approximate current values of the:
  - Pension assets: US\$ 10 billion;
  - Liabilities (comparable to the ABO liabilities in the FASB rules): US\$ 8.75 billion;
  - Salaries: US\$ 1 billion.

Note that this implies a funding level of about 115% and an “*asset / contribution leverage*” of 10. That is, if the contribution rate is defined as the ratio of the contributions to the salaries, then the *asset / contribution leverage* of 10 implies that  $\chi$  percent points of additional portfolio returns, amounts to  $10\chi$  percent points contribution rate.

- Indexation agreements:
 

The pension rights of the in-active members (= disabled, resigned and retired employees, and widows and widowers) are compensated for the growth of the cost of living as long as the funding level exceeds the minimally required value of 100%. If the indexation has fallen behind in years of underfunding, if possible the deficits are caught up once the violation of the minimum funding level has recovered.

- Funding agreements:

The funding agreements are characterised by two targets levels, i.e a maximum target of 135% and a minimum target of 115%. If the actual funding level is above the maximum target, the overflow is returned to the sponsor (resulting in potential negative contribution rates). If the funding level is below the minimum target, the contribution rate is increased, taking into account the restrictions that jumps of the contribution rate from one year to the next are limited to 2%-points of the salaries if the contribution exceeds a 12.5% of the salaries, and the contribution rate is not allowed to exceed a maximum level of 25%.

#### 4.2 Economic and Demographic Assumptions.

The consequences of asset / liability policies for the pension fund are evaluated against 500 scenarios with an horizon of 15 years of the economic environment of (Dutch) prices, wages, interest rates, and the total rate of return of an internationally diversified equity portfolio. The scenario generation is carried out using a Vector Auto Regressive time series model which is thoroughly described in [Boender, 1997a; Boender et al, 1997b]. The most important characteristics of the generated scenarios are depicted in Table 1. Note in particular that the risk premium of equities over bonds is equal to  $11.5\% - 7.8\% = 3.7\%$ , which is moderate in relation to the risk premium of 6% which has been observed over the period 1926/present in the USA (cf. [Benartzi and Thaler, 1993]), and that the volatility of equities, which will be used in the option pricing formula, is equal to 18.5%.

	Average values	Standard deviations
Prices	2.5%	2.4%
Wages	3.6%	3.0%
Interest rates (short term).	6.0%	1.8%
Interest rates (long term)	7.8%	5.0%
Total rate of return of equities	11.5%	18.5%

Table 1. Statistics of the 500 economic scenarios over the period 1996/2010.

For each economic scenario we use a Push- and Pull Markov model and the pension rules for the future development of the status and the relevant pension figures of the plan members. The transition probabilities of the Markov models are chosen such that survival, resignation and disablement remain in accordance with data over the previous 5 years, and the number people in each function category remains in accordance with the situation ultimo 1995. For additional information on the scenario generation of the liability figures, we again refer to the references cited above.

### 4.3 Basic Asset / Liability Policies: Contributions Rates and Funding Risks

Given the indexation- and funding agreements of the pension fund described in Section 4.1, we evaluated a number of asset allocations against the scenarios described in Section 4.2, assuming that the asset allocations in each year  $t$  ( $t=1996,\dots,2010$ ) of each scenario  $s$  ( $s=1,\dots,500$ ) are fully rebalanced to the initial portfolio. The results for 11 asset allocations, varying from 100% fixed income and 0% equities, to the allocation 0/100, are depicted in Figure 1. The colours in the pies characterise the asset allocation (light: fixed income; dark: equities). On the horizontal axis the resulting average contribution rates, defined as the contribution as percentage of the salaries averaged over all years  $t$  ( $t=1996,\dots,2010$ ) and all scenarios  $s$  ( $s=1,\dots,500$ ), are depicted. On the vertical axis the downside deviation of the funding levels defined above are depicted. Note that the high asset/contribution leverage of the pension fund referred to in Section 4.1., in the assumed economic environment leads to extremely low average contribution rates on the one hand, and relatively high funding risks on the other.

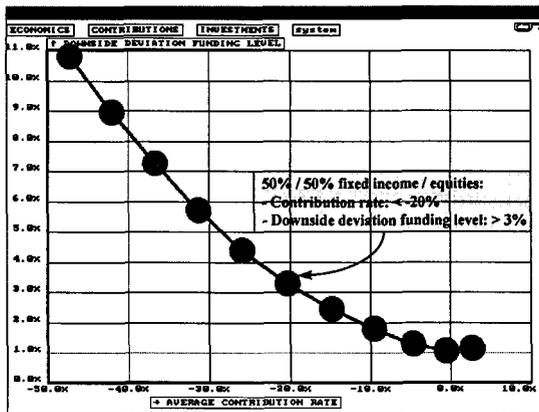


Figure 1. Basic asset / liability policies

## 5 Experimental Results

Now we turn to the experiments where put options are included in the asset / liability policies described in Section 4. The main results of our experiments are depicted in the Figures 2 and 3. Figure 2 depicts the results where for each asset / liability policy depicted in Figure 1, a path dependent dynamic put strategy is carried out which primo each year  $t$  of each scenario  $s$  aims to guarantee a funding level of 103% ultimo ( $t,s$ ) at a maximum put premium of 10% of the value of the equity portfolio. Figure 3 depicts the results for the roll-over long term version of this approach where primo ( $t,s$ ) a 5-years put is constructed to safeguard a funding level of 103% ultimo ( $t+5,s$ ) at a maximum put premium of 2% per year.

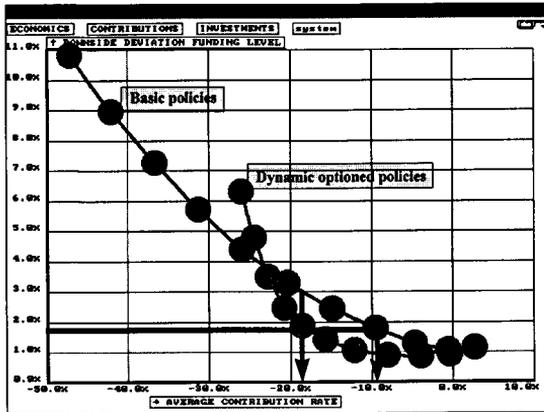


Figure 2. Dynamic optioned policies (1 year horizon)

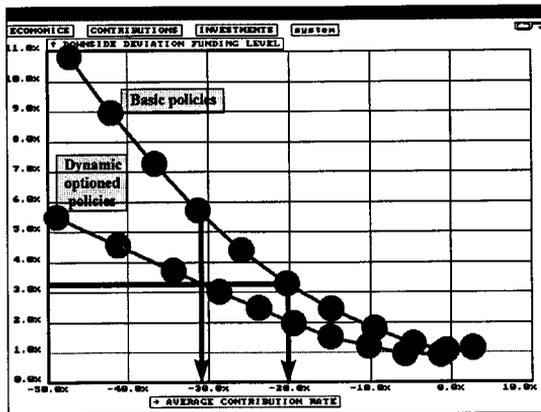


Figure 3. Dynamic optioned policies (5 years horizon)

It is immediate clear from the figures that the liability driven put strategies yield significant efficiency gains. That is, by applying these strategies either a contribution-neutral reduction of the downside deviation of the funding level can be accomplished, or a risk-neutral reduction of the contribution rate. Consider for example 70/30/- basic strategy in Figure 2 (i.e., 70% fixed income, 30% equities and no funding insurance). Given the indexation- and funding agreements, the assumptions about the economic environment imply a downside deviation of the funding level of about 2%. It can also be verified from Figure 2 that the 40/60/p1 strategy (=40/60 asset allocation and a 1-year dynamic put strategy) in exactly the same circumstances yields about the same downside funding risk, whereas this policy leads to 10 percentpoints reduction of the contribution rate (which with the basic policies could only be realised by a risk-increasing enlargement of the amount of equities from 30% to almost 50%).

The Figures 2 and 3 further display:

- For the **one-year** put strategies the efficiency gain reduces if the amount of equities in the portfolio is increased. This phenomenon which is clearly due to the ratio of the costs to fund the puts on the one hand, and the earned downside risk insurance on the other is not yet fully understood.
- The **longer term** put strategies perform even better than the one-year strategies. This is clearly due to the fact that the long duration of the liabilities of pension funds enables these organisations to profit from longer term insurance products, which evidently are relatively cheap in relation to their short-term counterparts.

In addition to the main results which are depicted in the Figures 2 and 3 we carried out many other experiments:

- First of all, any put strategy of class I in Section 3 (i.e. strategies which do not take into account the expected development of the liabilities) do not yield any efficiency gain in terms of contribution rate and the downside deviation of the funding level. Thus, if our results would be generally applicable, pension funds could not structurally gain from simple option strategies in an asset-only framework, and liability-driven put strategies like the ones that we designed above, are necessary to profit from the derivative instruments.
- Secondly, if the probability of underfunding is considered, rather than the downside deviation of the funding level, then the efficiency gains of the policies in Figure 2 and 3 reduce significantly. Thus, these put strategies provide a better insurance against the extent of underfunding, than to the probability that a underfunding occurs (note that our experiments also show that the efficiency gains of the liability driven put strategies vanish completely if the mean and standard deviation of portfolio return is considered, rather than the cost and risks of the pension fund).
- Finally, we also investigated the reduction of the efficiency gains for the case that the costs of the puts are higher than the ones which are obtained with the Black-Scholes formula with a volatility of 18.5%. The experiments indicated, for example, that the efficiency gain of the one-year put strategy which is carried out for the 50/50 asset-allocation in Figure 2, vanishes if the put is priced with a volatility of 23.5%, rather than 18.5%.

## 6 Conclusion

Our experiments verify that pension funds who measure risk by the second lower partial moment of the funding level can accomplish significant risk-neutral reductions of the contribution rate by carrying out liability driven path dependent put strategies. The experiments indicate, particularly if long term put strategies are carried out, that the efficiency gains can amount to the (risk-increasing) gains which would be accomplished by increasing the weight of equities in the portfolio at the expense of fixed income securities with 20 percent points.

However, much research remains to be done in this new field of research:

- First of all it has to be investigated to which extent our obtained results are generally applicable, and, how the results depend on other policy instruments such as the funding and indexation agreements.
- Secondly, expert opinion and dynamic optimization procedures should be employed to improve the put strategies described above. For example, should another type of put-policy be followed if the current funding level is either 75%, or 150%, or should the policy be adapted to the observed volatility ?
- Thirdly, much fundamental work remains to be carried out to integrate these approaches to insure insolvencies of liability organisations in the theory of finance, as well as consultancy efforts to integrate these approaches in the real-life management of assets and liabilities.

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