Chair for Risk Theory, Portfolio Management and Insurance
University of Mannheim
D-68131 MANNHEIM, GERMANY
Telephone: +49-621-292 5494
Facsimile: +49-621-292 5712
e-mail:risk@bwl.uni-mannheim.de

Risk Based Capital Allocation and Risk Adjusted Performance Management in Property/Liability-Insurance: A Risk Theoretical Framework

Abstract

Related to the current discussion of value-at-risk-based capital allocation and performance management (RAPM) for managing bank capital, a risk theoretical RAPM-approach for property/liability-insurance companies is presented. The paper discusses several central issues of a RAPM-approach: Virtual risk adjusted capital (VRAC) on the company level, return on risk adjusted capital (RORAC), risk based capital allocation to business segments, RAPM for business segments. RORAC-based premium principles are presented and discussed as well.

Keywords

Virtual risk adjusted capital, return on risk adjusted capital, risk based capital allocation.

Peter Albrecht

1. Introduction

Methods for risk based capital (RBC)-allocation and risk-adjusted performance management (RAPM)-techniques seem to be meanwhile well established in many large banks, cf. e.g. *Matten* (1996) for an introduction into this area. In case of insurance companies, the discussion on these topics has only just begun, cf. e.g. *Hooker et al.* (1996, chapter 5) or *Rigby* (1996). Insurance companies are different from banks (and industrial companies) in many respects and so a genuine approach is necessary. The present paper concentrates on property/liability-insurance and discusses several central issues of a RAPM-approach on a quantitative basis.

2. Preliminary Remarks

2.1 Virtual Risk Adjusted Capital (VRAC) and Physical Capital

There are many definitions of capital. In case of equity capital this could be either equity capital as published in the balance sheet or the former quantity including hidden reserves as well as share capital at book value or at market value. In case of insurance companies there is usually a regulation of capital requirements for solvency proposes, e.g. the E.U. solvency regulations or the U.S.-Risk Based Capital (RBC)-regulations. These regulations control the minimum amount of equity capital (in the form of solvency capital) of an insurance company with respect to the risk volume (measured according to specific rules). Solvency capital can be interpreted as an *external* RAC-conception. Risk based capital allocation is usually understood as a *notional* or pro forma allocation of capital cf. e.g. *Hooker et al.* (1996, 5.2.2) or *Matten* (1996, p. 32). This is distinct from an investment of capital in that no actual cash investment takes place. This is especially reasonable for insurance companies, where equity capital primarily serves as safety capital being invested in the capital markets and only a small amount being invested in physical assets of the insurance company. As in addition external versions of RBC depend on the respective regulatory environment and are criticized to measure risk properly, we propose to work with a conception of virtual risk based capital for allocation purposes. VRBC is an *internal* conception and may be entirely company-specific, when the insurance company has its own conception to calculate risk based capital properly. Different approaches to determine a VRAC for internal purposes are therefore possible, a recent example is *Skurnick/Grandisson* (1996), who claim their model (p. 304) to be in some ways superior to the U.S.-RBC formulas.

However, there must be some link between the physical capital of the insurance company (measured the one way or the other) and the VRAC. The link we propose is the overall target return on capital.

2.2 Target Return on Capital (TROC) and Target Return on Virtual Capital (TROVC)

There are different ways to specify a TROC, heavily depending on the conception of capital used. For example, management can prescribe a (minimum) TROC to be earned on the physical capital or the shareholders can require a (minimum) TROC on the market value of the share capital. TROC can be specified in a pre-tax or an after-tax version and it should be "risk adjusted" in the sense, that the more risky the insurance company's (total) business is, the higher the required overall TROC. If an (one-period) overall TROC r_T is specified, and relates to some physical capital (at book or market values) of amount C, then it is always possible to calculate an equivalent (in a certain sense) overall target return on virtual capital (TROVC) r_{VT} relating to some VRAC of amount VC, as we will show subsequently.

Let π denote the collective premium, S the accumulated collective claim amount, K the operating expenses and IP the overall investment profit of the insurance company for the period regarded, then

$$TP = \pi - S - K + IP \tag{1}$$

is the overall one-period result, the total profit of the insurance company. It is assumed that claim-dependent operating costs (e.g. claim regulation costs) are subsumed under S. As a consequence K is assumed to be a deterministic quantity, the same holds for π . S and IP, however, are regarded as stochastic quantities. If now r_T and C are speficed, then obviously C r_T is the absolute profit, which has to be earned in the respective period, a deterministic target profit for TP.

How can this quantity be converted for the purpose of RAPM? First of all, for the case of insurance companies we are convinced that only the *liabilities* should be controlled on the basis of a RAPM-approach. The investment profit should certainly be controlled, too. However, we think that the performance of the different asset categories should be measured and controlled on the basis of a benchmark conception (e.g. the asset category stocks relative to a stock index, etc.) This different treatment of assets and liabilities we propose has its motivation in the fact, that only in its core business, the liability side, the insurance company can act as a "market maker", i.e. is able to control the conditions of the business (at least to a certain extent), whereas on the asset side the insurance company typically is a "price taker", it only has limited influence on the condition of the business. So our primary objective is to control the underwriting profit UP of the company on the basis of a RAPM-approach. The conversion of C r_T intended now depends on the concept of underwriting profit used. Three concepts, which are differently narrow, can be used:

$$UP = \pi - S \tag{2a}$$

$$UP = \pi - S - K \qquad \text{or} \tag{2b}$$

$$UP = \pi - S - K - I_L \tag{2c}$$

In the last conception I_L denotes that part of the absolute investment profit, which is generated by the liabilities of the company (investment of premium and loss reserves or of generated cash flows). I_L is regarded as a deterministic quantity and can be factorized in the form I_L = ICL x i_L, where ICL is the (average) investment capital generated by the liabilities and i_L a deterministic *standard* return assumed for ICL, i.e. attributed to the investment of the liabilities (for example i_L could be a return which is assumed to be earned with high confidence over a longer period of time).

We now have the following relations between TP and UP in the various cases:

$$UP = TP + K - IP \ , \tag{3a}$$

$$UP = TP - IP \ \text{and} \tag{3b}$$

$$UP = TP - (IP - I_L) \ . \tag{3c}$$

Now as C r_T is an absolute deterministic target profit for the total profit TP and VC r_{VT} an absolute target profit for the underwriting profit UP is reasonable to assume the following relations in dependence of the conception of underwriting profit used:

$$VC\ r_{VT} = C\ r_T + K - E(IP) \ , \tag{4a}$$

$$VC\ r_{VT} = C\ r_T \ - E(IP) \ \text{and} \tag{4b}$$

$$VC\ r_{VT} = C\ r_T \ - [E(IP) - I_L] \ . \tag{4c}$$

Note that the relations (3a-c) are relations between quantities which are primarily stochastic, whereas the relations (4a-c) are relations between quantities which are all deterministic as we impose deterministic targets for stochastic profits. This is the reason that we work with E(IP), the *expected* total investment profit in (4a-c). However, any other deterministic conception of standard investment return can be used.

Implicit in (4a-c) is that we work with a fixed asset allocation structure in the planning period. If the asset allocation is changed, then E(IP) will change, too, which obviously must have consequences for VC r_{VT}. Basicly, however, the proposed approach only is a consequence of the fact that a certain part of the overall target profit will be earned on the asset side and the other part will be earned on the liability side. As we want to control the underwriting profits, we have to make some assumptions on the investment profit earned.

Summing up, on the basis of (4a-c) we can always convert an overall TROC r_T relating to some amount of physical capital to an equivalent TROVC r_{VT} relating to some virtual risk adjusted capital VRAC. As this approach obviously is independent of the conception for VRAC used, we can work with any reasonable conception for VRAC in the proposed way. In the following we will present a risk theoretical approach to determine a VRAC for performance management purposes.

3. RAPM on the Company Level

3.1 VRAC for the Purpose of Performance Management

First of all, as our objective only is to control the underwriting profits we are not concerned with the determination of an overall required risk based capital including the asset side, we only need to specify a conception for risk based capital for the liability side. Second our objective is not the control of solvency but performance management. To control solvency one would make sure that the physical capital available in the company does not fall below the (overall) risk based capital determined. To control performance in a risk-adjusted way one wants to assure, that a minimum risk-adjusted underwriting return is achieved. These are different purposes and so we would like to distinguish conceptions of (virtual) risk based capital for solvency purposes and conceptions of VRAC for the purpose of performance management. Both are clearly closely related, but in our view not identical. To work out the difference we in the following concentrate on the most narrow form of underwriting profit as given by (2a). All results can readily be translated to (2b) or (2c) as this only involves deterministic translations.

Central to the determination of risk based capital is a knowledge of the distribution F of S. This is a risk theoretical standard problem and we assume it to be solved, e.g. as done by *Skurnick/Grandisson* (1996). The risk theoretical standard approach to determine to required (minimum) risk based capital RBC for solvency purposes is to restrict the one-period ruin probability to a certain quantity ε. The determination of ε is (or should be) a central decision of the management of the insurance company, one popular approach in $\varepsilon = 0.01$, cf. e.g. *Skurnick/Grandisson* (1996, p. 297), and we assume that the problem of determining ε has been solved, too. Now, the technical condition for calculating RBC is:

$$P(UP < -RBC) = \varepsilon \;, \tag{5a}$$

which is - when using UP according to (2 a) - equivalent to

$$P(UP > RBC + \pi) = \varepsilon \; . \qquad \textbf{(5b)}$$

If $F_\varepsilon = F^{-1}(1\text{-}\varepsilon)$ denotes the $(1\text{-}\varepsilon)$-quantile of the accumulated claim distribution this in turn is equivalent to:

$$RBC = F_\varepsilon - \pi \quad . \qquad \textbf{(5c)}$$

The relations (5b) and (5c) clearly show that the risk based capital RBC and the premium π are *substitutive* quantities. The higher the premiums earned the less the required RBC when holding the safety level of the company constant. *Bühlmann* in the discussion of *Hooker et al.* (1996, p. 315) states this the following way: "There is a trade-off between capital and profitability". While we agree that the consideration of (historical) profitability is essential for solvency purposes, this amounts to a problem when considering performance management. Here we want to control the future profitability in a risk adjusted way and so VRAC should only contain risk elements and no element of profitability in order to construct an *unbiased* control quantitiy. As it is undisputed that $\pi(S) \geq E(S)$ should hold, we propose to set $\pi = E(S)$ in the determination of VRAC to obtain an unbiased quantity for the purpose of performance management. Summing up, the technical condition for the determination of VRAC is

$$P(\; S > VRAC + E(S)) = \varepsilon \quad , \qquad \textbf{(6a)}$$

being equivalent to

$$VRAC = F_\varepsilon - E(S) \quad . \qquad \textbf{(6b)}$$

Relation (6b) reveals, that the proposed VRAC measures risk as the distance between the $(1\text{-}\varepsilon)$-quantile of the distribution and its expected value. This measure of risk therefore corresponds to a measure of dangerousness of the accumulated claims distribution.

We in the following look at two examples. These examples are chosen for illustratory purposes only in that they result in closed formulas, which allows to check the consistency and validity of the proposed framework.

In case S follows a normal distribution, $S \sim N(E(S), \sigma(S))$, we have

$$F_\varepsilon = E(S) + N_\varepsilon \, \sigma(S) \quad , \tag{7a}$$

where N_ε denotes the $(1\text{-}\varepsilon)$ quantile of the standard normal distribution and the VRAC according to (6b) is given by

$$VRAC = N_\varepsilon \, \sigma(S) \quad . \tag{7b}$$

So the VRAC is proportional to the standard deviation of the accumulated claims distribution. This supports that VRAC is a pure risk measure.

In case of S following a Normal Power (NP)-distribution, cf. *Beard et al.* (1984, pp. 108 - 119), the $(1\text{-}\varepsilon)$-quantile of the NP-distribution is given by

$$F_\varepsilon = E(S) + N_\varepsilon \, \sigma(S) + \frac{1}{6} \, (N_\varepsilon^2 - 1) \, \frac{M_3(S)}{Var(S)} \quad , \tag{8a}$$

where $M_3(S) = E[(S-E(S))^3]$ denotes the third central moment of S and so we have

$$VRAC = N_\varepsilon \, \sigma(S) + \frac{1}{6} \, (N_\varepsilon^2 - 1) \, \frac{M_3(S)}{Var(S)} \quad . \tag{8b}$$

The central elements of the dangerousness of the NP-distribution therefore are contained in the calculated VRAC.

3.2 Risk Adjusted Performance Measurement

The core of RAPM is to assess the underwriting profit (of the company or of a business segment) relative to the attributed risk adjusted capital. From the principles of capital investment one knows that the absolute result of an investment alone is not informative enough, we have to judge it against the amount of capital invested, i.e. calculate a quantity of return. In addition from capital market theory we know that investors require higher returns for investments which bear higher risks. Risk adjusted performance measurement in banks therefore uses performance measures of the type

$$RORAC = \frac{(Absolute)\ Return}{RAC} \quad , \tag{9}$$

or related concepts as RAROC, etc., cf. *Matten* (1996, pp. 58 ff.). The investment return is risk adjusted by relating the (unadjusted) absolute return to a measure of risk based capital. An investment which requires more (notional) capital because of its higher riskiness must earn a higher return.

Translating this conception to our problem, the risk adjusted control of underwriting returns in property/liability-insurance, the respective measure would be

$$RORAC = \frac{UP}{VRAC} \quad , \tag{10}$$

using one of the versions (4a-c) for the underwriting profit and the conception of VRAC according to (6b). Using the most narrow version of UP, this means that we use the control-quantity

$$RORAC = \frac{\pi - S}{VRAC} \quad . \tag{11}$$

Looking at (11) in addition strengthens our argument to use for RAPM the VRAC according to (6b) and not the RBC for solvency purposes according to (5b), because in the latter case we would have versions of the premium π in the nominator and in the denominator of (11)! Besides that, we note that our approach is corresponding to the Value-at-Risk (VAR)-approach to RAPM discussed in *Matten* (1996, p. 60 ff.), because the VAR-approach is closely related (on the conceptual level) to the risk theoretic (one period) ruin approach, cf. *Albrecht et al.* (1996).

3.3 Risk Adjusted Performance Management

3.3.1 The Ex-ante Point of View: RORAC-Based Premium Principles

The objective of RAPM is to achieve a risk-adjusted minimum return r_{VT} from the underwriting operations (remember that when VRAC is given, then the relevant r_{VT} can be calculated on basis of the relations (4a-c)). As in an ex ante view the performance measure RORAC is a random variable, there are different ways to formalize this. Perhaps the most straightforward approach is to require

$$E(RORAC) \geq r_{VT} \quad . \tag{12a}$$

From (11) we equivalently obtain

$$\pi \geq E(S) + VRAC\, r_{VT} \quad . \tag{12b}$$

Using the VRAC-approach presented (3.1) this in term is equivalent to

$$\pi \geq E(S) + Z(S) r_{VT} \quad , \tag{12c}$$

where $Z(S) := F_\varepsilon - E(S)$ is the measure of dangerousness of the distribution of S according to (6b).

Relation (12c) corresponds to a risk theoretical premium principle of the form $\pi = E(S) + \lambda Z(S)$ where the safety loading is proportional to this measure of dangerousness and the loading factor λ is identical to the required TROVC. So in a certain sense the proposed approach is superior to the classical risk theoretical one in that it determines the loading factor λ and links it to return requirements of the management (according to the discussion in 2.2.) or even the capital markets, in case TROVC is calculated from a TROC related to requirements of the shareholders.

An other approach is possible. In generalization of the percentile principle one would require

$$P(RORAC \geq r_{VT}) = 1 - \varepsilon \tag{13a}$$

or equivalently

$$P(RORAC < r_{VT}) = \varepsilon \quad . \tag{13b}$$

With a controlled high probability the RORAC shall not fall below the TROVC. Using the narrow form (2a) for the underwriting profit, we equivalently obtain

$$P\left[\frac{\pi - S}{VRAC} < r_{VT}\right] = \varepsilon \quad . \tag{14a}$$

If we - hypothetically - choose $r_{VT} = -1$ (total loss of the VRAC) this specializes to

$$P(S > VRAC + \pi) = \varepsilon \quad , \tag{14b}$$

which reveals that the classical solvency condition is just a special case of the new approach! If we hypothetically choose $r_{VT} = 0$, then we obtain

$$P(S > \pi) = \varepsilon \quad , \tag{14c}$$

which is the traditional percentile principle! In general (14a) is equivalent to $P(S > \pi - VRAC\ r_{VT}) = \varepsilon$ and using again the $(1 - \varepsilon)$-quantile, we obtain

$$\pi = F_{\varepsilon} + VRAC r_{VT} \quad . \tag{15a}$$

Using the VRAC according to (6b) this can be cast into the following equivalent form (again $Z(S) := F_{\varepsilon} - E(S)$):

$$\begin{aligned} \pi &= E(S) + [F_{\varepsilon} - E(S)] + VRAC r_{VT} \\ &= E(S) + Z(S)(1 + r_{VT}) \quad . \end{aligned} \tag{15b}$$

This premium principle again has a safety loading proportional to the measure of dangerousness $Z(S)$ with loading factor now amounting to $\lambda = 1 + r_{VT}$, mirroring the stronger requirement of (13a) compared to (12a).

At last we want to note, that if the above approaches are to applied in practice one has to empirically identify the central quantities, like $E(S)$, $Var(S)$, $Z(S)$ and also VRAC!

3.3.2 The Ex post-Point of View: Controlling the Realized RORAC

In the ex post-point of view one has to control whether the realized return on risk adjusted capital (RRORAC) meets the requirements, i.e. does exceed the TROVC r_{VT}, for one period or over a number of past periods. RRORAC has to be based on the average premium income $\bar{\pi}$ over the periods and an estimate (including claims averages, IBNR-estimation and a treatment of large claims) $\hat{E}(S)$ of $E(S)$. VRAC has to be chosen as in the planning stage, cf. 3.3.2. It also has to be estimated, but beforehand. The RRORAC therefore is given by

$$RRORAC = \frac{\bar{\pi} - \hat{E}(S)}{VRAC} \quad . \tag{16}$$

The most straight forward control criterion would be

$$RRORAC \geq r_{VT} \quad , \tag{17}$$

which is just the ex post-version of (12a)! For an ex post-version of (14a) we propose to use the correspnding ex post-version of (15b).

4. RAPM on the Level of Business Segments: Capital Allocation

4.1 Requirements of Capital Allocation

Let the total underwriting business of the insurance company be made up of $i = 1, \ldots, n$ business segments with corresponding premium income π_i, aggregated claims amount S_i, operating expenses K_i, attributed investment income I_L (i) generated by the segments liabilites and underwriting profits UP_i. No assumption that the S_i are independent or uncorrelated will be made, on the contrary, the dependence of the S_i is at the heart of the problem to be solved.

The RAPM of the business segments again will be based on performance measures of the RORAC-type (10):

$$RORAC_i = \frac{UP_i}{VRAC_i} \quad , \qquad i = 1,\ldots,n. \tag{18a}$$

In the following we will restrict to the most narrow form for the underwriting profit of the segments, i.e. consider only segment-RORACs of the form

$$RORAC_i = \frac{\pi_i - S_i}{VRAC_i} \quad . \tag{18b}$$

The RAPM of the underwriting profits of the business segments has to be done in a consistent way with the RAPM on the company level. This is done by choosing an identical (risk adjusted) TROVC r_{VT} for the company as well as for all business segments on one hand and by allocating the total VRAC in a way, that $\sum_{i=1}^{n} VRAC_i = VRAC$. This means, that for the problem of capital allocation we have to determine factors of proportionality (allocation factors) x_i with $0 \leq x_i \leq 1$ and $\sum_{i=1}^{n} x_i = 1$ in order to determine $VRAC_i$ according to

$$VRAC_i = VRAC\, x_i \quad , \qquad i = 1,\ldots,n. \tag{19}$$

That the problem of capital allocation is a non-trivial one, is caused by the (substantial) reduction-of-risk effect when aggregating the different business segments to the total portfolio of the company. If we would calcutate the $VRAC_i$ according to the same principle (6a) as the total VRAC, but in an isolated way for each of the segments i = 1, ..., n (i.e. neglecting in each case the existence of the other segments), we would obtain

$$VRAC_i^* = F_\varepsilon^i - E(S_i) \quad , \tag{20}$$

where F_ε^i denotes the (1-ε)-quantile of the distribution of S_i. Now as a rule we have $\sum F_\varepsilon^i > F_\varepsilon$ and therefore $\sum VRAC_i^* > VRAC$. The VRAC on the company level is not composed in an additive way from the isolated VRACs of its segments (this would make the capital allocation a trivial problem). This means that we have to perform something like a risk adequate allocation of the total VRAC to determine the $VRAC_i$. The basic idea is the following. As the total VRAC for performance management purposes is a risk measure or at least determined on the basis of a measure of risk, the allocation (the subdivision of VRAC) should be done *risk proportional*, i.e. on the basis of a measure of risk. As the total risk of the portfolio in general (i.e. for an arbitrary distribution function F) will not be linear in the corresponding risks for the segments (e.g. the F_i) on one hand and as any allocation requires such a linear relation, the solution *will not be unique* and thus to some extent arbitrary. Different approaches for capital allocation will be based on *different linear approximations* of risk. Although the allocation will be to some extent abitrary this does not mean that there are no reasonable approaches. One has to consider different criteria of goodness or quality and to judge possible allocations according to these criteria. Possible criteria are the following:

1) Consistency with the determination of VRAC at the company level;
2) Quality of the risk measure used;
3) Consideration of the stochastic dependencies between the business segments;
4) Possibility of practical implementation of the approach for RAPM.

In addition, the chosen capital allocation rule may depend on the purpose of the analysis, i.e. the decisions which it should support.

In the following we therefore only are able to present some possible rules for allocation and try to judge them according the above criteria.

4.2 Volatility-Based Capital Allocation

4.2.1 Stochastically Independent Segments

In the following let denote $\sigma^2 = \mathrm{Var}(S)$ resp. $\sigma_i^2 = \mathrm{Var}(S_i)$ the variance of the accumulated claims on the company level resp. for the business segments and σ resp. σ_i the corresponding standard deviations. In case of stochastically independent S_i we have an additive relation for the variances, $\sigma^2 = \sigma_1^2 + \ldots + \sigma_n^2$ and a non-linear relationship between the standard deviations, $\sigma = [\sigma_1^2 + \ldots + \sigma_n^2]^{1/2}$.

An allocation of the VRAC being variance-proportional would result in the following choice of the allocation factors x_i according to (19):

$$x_i = \frac{\sigma_i^2}{\sigma^2} = \frac{\sigma_i^2}{(\sigma_1^2 + \ldots + \sigma_n^2)} \quad . \tag{21}$$

While it is *not* possible to decompose σ itself in a linear way an allocation which is in some sense standard deviation-proportional would be:

$$x_i = \frac{\sigma_i}{(\sigma_1 + \ldots + \sigma_n)} \quad . \tag{22}$$

However, this is not an allocation based on the decomposition of a measure of total risk, here $\sigma(S)$. We regard this as a weakness and therefore would prefer the variance-proportional allocation factors according to (21).

Variance and standard deviation are risk measures, which are volatility-based, they measure deviations from the expected value E(S) in both directions. This means especially that realizations of S below E(S) are regarded as risky, too, although this is favourable for the insurance company. This is a disadvantage compared to shortfall-based risk measures, which will be discussed later. This disadvantage will be important only for asymmetrical distributions, but these are the relevant ones for property/liability insurers.

Let us take a look at the examples in 3.1 based on distributional assumptions. In case of a normal distribution we obtain VRAC = $N_\varepsilon\ \sigma(S)$ according to (7b). As already stated before $\sigma(S)$ is composed of the $\sigma(S_i)$ in a non-linear way, so even in the most simple case of a normal distribution there is *no possibility* to achieve an *exact* linear decomposition of the total VRAC! Looking at the VRAC (8b) in case of the NP-distribution, one sees that an allocation would in addition require a consideration of the skewness or the third central moment, which, however, would cause some troubles for stochastically dependent S_i. In general, therefore a risk based capital allocation *will depend on the distributional assumption chosen.*

4.2.2 Stochastically Dependent Segments

In case of stochastic dependency we have the folllowing decomposition for the total variance:

$$\begin{aligned} Var(S) &= \sum_{i=1}^{n} \sum_{j=1}^{n} Cov(S_i,\ S_j) \\ &= \sum_{i=1}^{n} [Var(S_i) + \sum_{j \neq i} Cov(S_i, S_j)] \quad . \end{aligned} \tag{23}$$

The second decomposition shows, which part of the total variance is to be attributed to a single business segment, namely its own variance and its covariances with all other segements.

A variance-proportional decomposition of VRAC would result in the following allocation factors:

$$x_i = \varphi_i = \frac{\sigma_i^2 + \sum_{j \neq i} \sigma_{ij}}{\sigma^2} = \frac{\sum_{j=1} \sigma_{ij}}{\sigma^2} \tag{24}$$

where σ_{ij} = Cov(S_i, S_j) and $\sigma_{ii} = \sigma_i^2$. In the Value-at-Risk literature the quantities according to (24) are called (asset´s) φ (phi), cf. *Beckström/Campbell* (1995, p. 81).

The φ_i measure the relative contribution of each segment to the total accumulated claims variance. Note, that the values are not constant, they depend on the relative size of the business segments (the "liability allocation"). If the liability allocation should change, for example by increasing the volume of a certain segment, the φ-values will change too.

A corresponding allocation based on factors proportional to the standard deviation would lead to the following allocation factors:

$$x_i = \frac{\sqrt{\sum_{j=1}^{n} \sigma_{ij}}}{\sum_{i=1}^{n}\left[\sqrt{\sum_{j=1}^{n} \sigma_{ij}}\right]} \quad . \tag{25}$$

But again this is not based on the decomposition of a measure of the total risk of the insurance company.

As the capital allocations proposed in this chapter heavily rely on the correlations between the segments, in practical applications of this concept the empirical correlations should be examined for their stability and a sensitivity analysis should be performed using alternative amounts for the correlations used.

4.3 Beta-Based Capital Allocation

A risk measure central to capital market theory is $\beta = \mathrm{Cov}(R, R_M)/\mathrm{Var}\,(R_M)$, where R denotes the return of a security and R_M the return on a market index. The beta-coefficient is a measure for the systematic risk of the security, i.e. its co-movement with the market. In connection with capital allocation and with applications of the shareholder value-approach the use of beta is very popular, too, because it can help to determine divisional required returns and divisional cost of capital, cf. e.g. *Van Horne* (1992, chapters 8, 9). Beta resp. the CAPM is also used in financial pricing models for insurance companies, cf e.g. *Cummins* (1990).

Using "underwriting betas" for the business segments we can use a beta-approach for our problem, too. We will just present a very simple example how this could be done in principle. Defining an underwriting return $R = 1 - \frac{S}{\pi}$ on the company level and underwriting returns $R_i = 1 - \frac{S_i}{\pi_i}$ (i = 1, ..., n) for the business segments, we have the relation

$$R = \sum_{i=1}^{n} R_i \frac{\pi_i}{\pi} \quad . \tag{26}$$

Let I_M denote the return on a chosen market index we obtain

$$\beta(R,\ I_M) = \sum_{i=1}^{n} \beta(R_i,\ I_M) \frac{\pi_i}{\pi} \quad . \tag{27}$$

This is a linear decomposition of risk and so we could use the allocation factors:

$$x_i = \frac{\beta(R_i,\ I_M)\ \pi_i}{\beta(R,\ I_M)\ \pi} \quad . \tag{28}$$

However, we personally are very sceptical in using this approach, resp. in using underwriting betas at all, cf. *Albrecht et al.* (1991, chapter 2), in practial applications. Usually I_M is choosen to be a stock market index and it is hard to grasp that there are close connections between the development of a stock market index and the development of business segments of an insurance company. As *Cummins/Harrington* (1985) conclude, the use of underwriting betas can lead to inaccuracies because estimation errors can be quite significant.

4.4 Shortfall-Based Capital Allocation

Shortfall-based risk measures, cf. e.g. *Albrecht* (1994 a), conceptualize risk as the danger to fall below a specific target return z. Transferring this conception to insurance risk, cf. *Albrecht* (1994 b), this means that we are concerned with the danger that the underwriting profit falls below a certain target. Working with the narrow version (2a) of underwriting profit, setting $\pi = E(S)$ as discussed in 3.1 and choosing as target $z = 0$ (non-negative underwriting profit) the risk measure shortfall probability would now be

$$SP_0(S) = P(\pi - S < 0) = P(S > E(S)) \quad . \tag{29}$$

Risk is conceived as exceeding the expected claims value. Corresponding allocation factors could be

$$x_i = \frac{P(S_i > E(S_i))}{\sum_{j=1}^{n} P(S_j > E(S_j))} \quad . \tag{30}$$

Under the same assumptions the shortfall expectation would be given by

$$SE_0(S) = \int_{E(S)}^{\infty} (s - E(S))\, f(s)\, ds = \mu_+(S) \quad , \tag{31}$$

i.e. corresponds to the average excess of the claims over their expected value. An corresponding allocation rule could be

$$x_i = \frac{\mu_+(S_i)}{\sum_{j=1}^{n} \mu_+(S_j)} \quad . \tag{32}$$

Another related allocation rule could be based on the semivariance σ_+^2 (S) of the claims distribution.

Note, however, that the preceding allocation rules are *not* based - as was the case with the standard deviation - proportional factors (22) resp. (25) - on a linear decomposition of the corresponding measure of the total risk of the company, which would be here P(S > E(S)) resp. μ_+ (S) into linear combinations of terms involving the business segment risks, here P(S_i > E(S_i)) resp. μ_+ (S_i). This is because how to obtain such a linear decomposition remains unclear. In addition, in the case of correlated segments, the correlations between the S_i are not recognized by the proposed allocation rules. We regard this as central weaknesses.

4.5 Quantile-Based Capital Allocation

Quantile-base capital allocation is directly related to the determination (6b) of the total VRAC proposed in this contribution. Corresponding allocation factors could be based on the $VRAC_i^*$ according to (20), i.e.:

$$x_i = \frac{F_\varepsilon^i - E(S_i)}{\sum_{j=1}^{n} F_\varepsilon^j - E(S_j)} \quad . \tag{33}$$

Once more, however, this allocation rule is not based on a decomposition of VRAC = F_ε - E(S) itself and does not consider possible correlations between the segments.

4.6 Incremental and Marginal Analyses of VRAC

Hooker et al. (1996, p. 294) consider a concept of *marginal* RBC, where each business unit is allocated the difference between

- the capital required by the company *with* that business unit; and
- the capital required by the company *without* that business unit.

As it is done in *Litterman* (1996, footnote 12, p. 75) we would prefer to call this approach an incremental one. Putting the approach into quantitative terms, the allocation would be based on VRAC(S) - VRAC(S-S_i). As it is unclear in general, whether these terms sum up to VRAC(S), the corresponding allocation factors could be

$$x_i = \frac{VRAC(S) - VRAC(S - S_i)}{\sum_{j=1}^{n} [VRAC(S) - VRAC(S - S_j)]} . \qquad \textbf{(34)}$$

A variation of this, which is easier to analyze, is a capital allocation based on incremental variance leading to the following allocation factors:

$$x_i = \frac{Var(S) - Var(S - S_i)}{\sum_{j=1}^{n} [Var(S) - Var(S - S_j)]} . \qquad \textbf{(35)}$$

In case of uncorrelated segments we obviously have $\sum_{j=1}^{n} [Var(S) - Var(S - S_j)] = Var(S)$

and Var(S) - Var(S - S_i) = Var(S_i). Therefore (35) is identical to the variance-based allocation rule (21). However, as is easily seen e.g. for n = 2, in case of correlated segments the denominator of (35) does not any longer sum up to Var(S) and so allocation (35) - and as well (34) in general - does not properly rely on a linear decomposition of total risk into the contributions of the different segments.

The nominators in (34) reps. (35) give an indication, of how the total VRAC resp. Var (S) will change, if a certain business segment is "closed", but this is a different situation because then a VRAC of a different amount has to be allocated to the (remaining) segments. So in our view the preceding incremental approach is better suited for *risk analysis* than for capital allocation. In addition, removing a complete business segment is only one (and an extreme one) decision alternative. Other decision alternatives could be to reduce or extend the business segments to some extent. The consequences of this could be analyzed on the basis of a marginal analysis, based on derivatives, as it is done in *Litterman* (1996, pp. 59 - 65). However, contrary to Litterman's statements, we do not see how this easily can be done, as the VRAC $F^{-1}(1-\varepsilon) - E(S)$ according to (6b) - being a quantile value as the value-at-risk-changes in a very complex way, when there are changes in the liability portfolio composition, especially when the segments are correlated, because all changes have immediate influences on F, the accumulated claims distribution. In addition, it remains unclear whether a marginal analysis gives a good approximation to the change in VRAC, when there are non-marginal changes in the liability portfolio.

4.7 Segment-Specific RORAC-Based Premium Principles

Based on the allocation of the total VRAC to the business segments the premium principles (12b) resp. (15a) can be decomposed, too. This leads to segment specific RORAC-based premiums, which are consistent with the company level in that they sum up to the total required premium, because $\Sigma\, VRAC_i = VRAC$. This approach is a solution to the problem of premium calculation from "top down", cf. *Bühlmann* (1988). E.g. the segment specific version of (12b) would be

$$\pi_i = E(S_i) + VRAC_i r_{VT} \quad . \tag{36}$$

Obviously $\Sigma\pi_i = E(S) + VRAC\; r_{VT} = \pi$ according to (12b).

References

Albrecht, P. (1991): Kapitalmarkttheoretische Fundierung der Versicherung, Zeitschrift für die gesamte Versicherungswissenschaft 80, 499 - 530.

Albrecht, P. (1994 a): Shortfall Returns and Shortfall Risks, in: Actuarial Approach for Financial Risks, 4th AFIR International Colloquium, Orlando, Vol. I, 87 - 110.

Albrecht, P. (1994 b): Dimensionen des versicherungstechnischen Risikos, in: *Hesberg, D., M. Nell, W. Schott* (eds.): Risiko, Versicherung, Markt, Verlag Versicherungswirtschaft, Karlsruhe, 325 - 339.

Albrecht, P., H.W.F. Bährle, A. König (1996): Value-at-Risk: A Risk Theoretical Perspective with Focus on Applications in the Insurance Industry, in: *Albrecht, P.* (ed.): Aktuarielle Ansätze für Finanz-Risiken, AFIR 1996, Verlag Versicherungswirtschaft, Karlsruhe, Vol. I, 3 - 24.

Beard, R.E., T. Pentikäinen, E. Pesonen (1984): Risk Theory, 3rd ed., London.

Beckström, R.A., A.R. Campbell (1995): The Future of Firm-Wide Risk Management, in: *Beckström, R.A., A.R. Campbell* (eds.): An Introduction to VAR, Palo Alto, 77 - 94.

Bühlmann, H. (1985): Premium Calculation from Top Down, ASTIN Bulletin 15, 89 - 102.

Cummins, J.D. (1990): Asset Pricing Models and Insurance Ratemaking, ASTIN Bulletin 20, 125 - 166.

Cummins, J.D., Harrington (1985): Property-Liability Insurance Regulation: Estimation of Underwriting Betas Using Quarterly Profit Data, Journal of Risk and Insurance 52, 16 - 43.

Hooker, N.D. et al. (1996): Risk-Based Capital in General Insurance, British Actuarial Journal 2, 265 - 323 (incl. discussion).

Litterman, R. (1996): Hot Spots™ and Hedges, The Journal of Portfolio Management, Special Issue 1996, 52 - 75.

Matten, Ch. (1996): Managing Bank Capital: Capital Allocation and Performance Measurement, J. Wiley & Sons, Chichester et al.

Rigby, B. (1996): New Models, The Review, June 1996.

Skurnick, D., M. Grandisson (1996): Multi-line risk measurement, XXVII-th ASTIN-Colloquium, Copenhagen, 292 - 309.

Van Horne, J.C. (1992): Financial Mangement and Policy, 9th ed., Prentice-Hall, Englewood Cliffs, N.J.