A METHODOLOGY FOR THE MODELLING OF INTEREST RATES AND OTHER ECONOMIC VARIABLES WITH REFERENCE TO THE MONEY AND CAPITAL MARKETS OF SOUTH AFRICA

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Abstract

This article describes a methodology for determining an appropriate structure for time-series models of short-term and long-term interest rates relative to inflation and other economic variables required for the modelling of the assets and liabilities of a financial institution.

The use of the methodology is illustrated with reference to data relating to the South African economy.

Keywords

Stochastic models, interest-bearing securities, equities

Introduction

The modelling of the cash flows generated by the assets and liabilities of a financial institution is normally done (e.g. Wilkie (1986), Carter (1991), Claassen (1991) and Tilley (1990)) by means of time series using autoregressive integrated moving average (ARIMA) processes with transfer-function-noise equations of the following form:

$$Y_t = \sum_{j=1}^{J} \frac{\omega_{j,0} - \sum_{i=1}^{s_j} \omega_{j,i} . B^i}{1 - \sum_{i=1}^{r_j} \delta_{j,i} . B^i} X_{j,t} + \frac{1 - \sum_{i=1}^{q} \theta_i . B^i}{1 - \sum_{i=1}^{p} \phi_i . B^i} \eta_t$$

where:

Y_t is the variable being modelled (i.e. the "output variable"), after differencing if necessary as explained below, and after subtraction of the mean of the (differenced) variable;

$X_{j,t}$ is the value of the jth input variable at time t;

η_t is a "white-noise" variable (i.e. a zero-mean variable that is not autocorrelated);

B is the backwards operator with respect to time; i.e. $B^i X_{j,t} = X_{j,t-i}$.

If the output variable is independent of any of the other variables, there may be no input variable, in which case the output variable is merely a function of the white-noise variable. It may be found that a particular variable is a non-stationary time series. In that case, the variable needs to be differenced d times, where d is the lowest integer such that $(1-B)^d X_t$ is stationary.

A univariate process for a variable that has been differenced d times and has autoregressive and moving average parameters p and q as shown in the above equation is referred to as an ARIMA(p,d,q) process.

It may be found necessary to allow for time-varying values of some of the parameters, in particular the standard deviation of the white-noise variable (in which case the model is referred to as an autoregressive conditional heteroscedastic (ARCH) model or one of its variants - cf. Hua (1994) and Harris (1994)).

The model may then be used to simulate the cash flows generated by the assets and liabilities of the financial institution up to a given time horizon and the values of those assets and liabilities at that time horizon. Such simulations, based on initial observed values of the input variables and generated values of the white-noise variables, may be repeated a large number of times so as to facilitate decision-making, particularly with regard to the apportionment of assets between various classes.

In order to develop such models, an objective framework must be established within which to answer the following questions.

(1) Which variables should be modelled?

(2) What time-intervals should be used?

(3) What function of each variable should be modelled?

(4) For each output variable:

(a) which variables (if any) should be used as input variables?

(b) how will the values of p, d and q be determined and, for each input variable, how should the values of r_j and s_j be determined?

(c) how should the values of $\{\theta_i \mid i \in [1, q]\}$ and $\{\phi_i \mid i \in [1, p]\}$ be determined and, for each input variable, how should the values of $\{\omega_{j,i} \mid i \in [0, s_j]\}$ and $\{\delta_{j,i} \mid i \in [1, r_j]\}$ be determined?

(d) what distribution should be assumed for the white-noise variable?

(5) What formulae should be adopted in order to use the model of the variables to simulate the cash flows generated by the assets and liabilities of the financial institution up to a given time horizon and the values of those assets and liabilities at that time horizon?

The purpose of this article is to address questions 1, 2, 3 and 4(a) above.

Questions 4(b) and (c), normally referred to as "identification" and "estimation" respectively, are dealt with by the standard texts on time-series modelling (e.g. Box and Jenkins (1970) and Granger and Newbold (1977)), although the use of standard time-series modelling procedures has certain draw-backs in actuarial applications, which are discussed in this article.

Question 4(d) may be addressed by analysing the distribution of the residuals (e.g. Wilkie (1984) and Thomson (1994)). However, it should be noted that, even if the white-noise variables are normally distributed, the observed residuals may be non-normal. Under these circumstances it is perhaps unnecessary to attempt to fit an alternative distribution where the residuals are non-normal. On the other hand, if the departure from normality is significant, it may mean that the model has been incorrectly identified. It appears that further research is needed here. In the mean time the fitting of non-normal distributions appears to imply a degree of sophistication that cannot be justified on sound theoretical foundations. Another alternative would be to use the bootstrap method, which involves using the sample distribution of the residuals as the distribution of the white-noise variable.

Question 5 is outside the scope of this article.

To a certain extent the answers to the questions are interrelated and the process of developing a model may therefore lead one down many blind alleys, which it is hoped this paper will help to avoid.

The model developed by Wilkie (1984, 1986 and 1987) with reference to British retail price inflation, British equities and British government securities, generally referred to as the "Wilkie model", has been applied to other economic environments (Wilkie 1994a and 1994b), but Carter (1991) found it unsuitable for Australia and Claassen and Huber (1992), Claassen (1993) and Thomson (1994) found it unsuitable for South Africa. It is also questionable whether the structure of the Wilkie model, which was developed with reference to British data, is optimal for all the other economic environments to which it has been applied.

The Wilkie model has also been extended to include exchange rates (Wilkie (1994b)) and British treasury bills (Ong (1994)). It is questionable whether the manner in which such additional variables have been included is optimal - in particular, whether those variables should not have been included as input variables for some of the variables initially modelled.

This article suggests a methodology that could enable practitioners to develop models suitable for their own specific purposes. The methodology, some of which is derived from Wilkie (1984, 1986 and 1987), is largely applied by Thomson (1994) and the results of that application are referred to in this article.

(1) Which variables should be modelled?

In the first instance, this question is interrelated with question (5); sufficient variables should be modelled to enable the assets and liabilities of the financial institution to be simulated in such a way as to facilitate decision-making. For example, if the simulations are being undertaken in order to facilitate decision-making with regard to the apportionment of assets between various classes using criteria relating to the liabilities, sufficient variables should be modelled for each class of assets, and for the liabilities, to enable the sensitivity of the simulated values to alternative allocations to be tested. For this purpose, a "class of assets" would be defined with reference to the decisions to be made. For example, if the purpose of the exercise is to determine asset apportionment for the purpose of defining a benchmark for investment performance measurement, the classes of assets to be separately modelled should correspond to the classes of assets that will be separately measured.

For long-term interest-bearing securities, if an index is used, it is necessary also to model the payments of coupon on the constituents of that index. In practice it is more convenient to model the yield to redemption on a notional gilt of a specified outstanding duration. In running simulations of cash flows it may then be assumed that, at the end of each time interval, the amount already invested in that gilt is realized at market value for the reduced duration at the new yield to redemption, and the amount to be invested in gilts of the full outstanding duration is invested in a par stock at that yield. More than one specified outstanding duration may be used if necessary.

Alternatives to yields to redemption would be the yields on zero-coupon bonds (namely, spot rates derivable from the yield curve), or forward rates that are derivable from the spot rates (Tilley (1990)). But either of these alternatives would necessitate modelling values for the full range of outstanding terms to redemption.

For money-market instruments an index may be used (as in Thomson (1994)) or a rate of interest may be defined with reference to a specific instrument (such as the discount rate on three-month treasury bills, which was used by Ong (1994)).

For equities, one would need to model dividends for the cash flow. For the investment of future cash flows one would need to model market prices. For the value at the time horizon one would need to model market prices if equities are to be taken at market value; alternatively, one may wish to model dividends or earnings in order to place a value on the equities at the time horizon. In order to obviate unnecessary modelling, one might wish to model any two of: an index of dividends, dividend yields, and an index of market prices. The choice appears to be immaterial. The Maturity Guarantees Working Party (MGWP) (1980) and Wilkie (1984, 1986 and 1987) uses dividends and dividend yields and that practice is followed by Thomson (1994). Carter (1991) uses dividend yields and an index of market prices.

If the purpose of the modelling of equities is to include them as a specific class in an investment performance benchmark, one would define equities to comprise the same index-constituents as those which will be used for the benchmark.

For property, similar considerations would apply. If decisions are going to be made with reference to a particular index (e.g. an index of property shares, or of unitized property funds, or a direct portfolio-based or barometer index), the relevant index should be modelled. It should be borne in mind, however, that:

(1) property shares may be geared, and some of the cash flows may arise from development and trading, thus restricting their appropriateness as a proxy for property;

(2) the market values of property shares are subject to sentiment prevailing in the equity market, which is not necessarily a good reflection of sentiment prevailing in the property market;

(3) the values of property fund units and of property indices depend on surveyor-determined property values, which may be expected to be less volatile than actual sale prices;

(4) the tax treatment of indirect property instruments may be different from that of direct property holdings;

(5) it may not be possible to diversify an individual institution's property portfolio to the extent implicit in these proxies;

(6) the current state of asset-liability modelling does not permit any allowance for the risks arising from the lack of liquidity associated with direct property holdings;

(7) portfolio-based indices generally take some time to calculate;

(8) barometer indices are unsuitable for performance measurement, since the institution could not closely match its movement with an actual portfolio of property holdings.

Other variables required for the modelling of the assets might include yields on index-linked bonds (Aziz and Prisman (1994)), exchange rates (Wilkie (1994a and 1994b)), and foreign investment instruments.

For the liabilities it is necessary to identify the major variables on which future cash-flows will depend. For example, for a defined-benefit pension scheme, wage inflation (which affects liabilities for active members) and price inflation (on which pension increases are often based) are of importance. Other variables relate to demographic effects. Because these variables have a lesser effect on the finances of such a scheme, and because they are not as strongly correlated with the variables used for simulating asset cash flows, they are generally modelled deterministically; as pointed out by Clark (1992), it is likely that the uncertainties in the economic elements of the basis will far outweigh those connected with the decrements.

All the variables discussed above are endogenous variables; each of them is required for the purposes of modelling the assets and liabilities of a financial institution. It is often asked whether exogenous variables should be modelled, so as to provide additional input variables for the simulation process.

Only if the values of a particular exogenous variable (or its residuals) are strongly correlated with subsequent values of any of the endogenous variables will it be possible for it to have any significant effect on the model, but:

(1) unless the current and recent values of the exogenous variable are significantly different from their expected values the effect will not be significant;

(3) depending on the lags included in the relevant transfer functions, the effect may be short-lived;

(4) the effect on the differences between the values of the endogenous variables, which may be more important than the values themselves, may be relatively insignificant.

In view of the above considerations and the complexity that would be added to the model by the addition of all the exogenous variables that might *prima facie* be considered appropriate, the inclusion of any exogenous variable requires careful justification. This may be a worthwhile area for further research.

A similar question relates to the modelling of subclasses of a particular asset class. For similar reasons, it is not considered worthwhile, for example, to model various subclasses of equities separately unless it is intended to treat them separately for the purposes of asset allocation benchmarks and investment performance measurement. Otherwise, the same arguments apply to subdivisions as to exogenous variables.

One may be restricted by one's choice of variables by the quality and quantity of data available. Granger and Newbold (1977) express lack of confidence of successful model identification with much less than 45 to 50 observations of each variable. However, simple models have been constructed with less data. The data should cover a period at least twice the intended simulation period (e.g. Geoghegan et al (1992)). But just as a model structure developed with reference to a particular economic environment may be found to be unsuitable for another, so - as is pointed out by Kitts (1990) and Geoghegan et al (1992) - structures may change over time. Thus, for periods exceeding, say, 50 years, or where there have been noticeable changes, it is advisable to model a more recent period as well as the full period, as is done by Wilkie (1991 and 1984).

(2) **What time intervals should be used?**

In general the option would be between quarterly and annual intervals. Monthly intervals would only be considered for relatively short-term models. In this section, it is assumed that the option is between quarterly and annual intervals, but by and large the criteria could be similarly applied to the option between monthly and quarterly intervals.

In deciding this question the following matters need to be considered.

Advantages of annual intervals

(1) The use of quarterly data in the development of the model tends to accentuate the short-term relationships at the expense of longer-term relationships. Wilkie (1992) points out that, "where the discrete models are equivalent to a continuous diffusion process, it is often the case that they are indistinguishable from a random walk when the observation period is sufficiently short. The "noise" overwhelms the signal. This may explain why observations over too short a period have not observed the longer term stabilities represented in the models of Wilkie and Tilley."

(2) If the liabilities depend on demographic models that are based on annual age intervals, it is simpler to use annual intervals for the simulations.

(3) Revenue accounts are normally prepared on an annual basis, so that simulation on a quarterly basis would not be comparable.

(4) A quarterly model may be more complex both in its specification and in its application.

(5) Dividend yields reflect dividends paid during a particular year. In some countries dividends are generally payable half-yearly. In those countries it

would be time-consuming to determine a measure of dividends paid during a particular quarter. Carter (1991) uses an approximate method to obtain quarterly dividends.

(6) For some variables, (e.g. direct property indices) data may be available only on a yearly basis.

(7) Seasonal effects would have to be allowed for if quarterly intervals were used, adding considerably to the complexity of the model.

(8) For some variables, quarterly data may exhibit relatively high kurtosis (Harris (1994).

(9) Quarterly data are more likely to exhibit ARCH effects.

Advantages of quarterly intervals
(1) More data points are used, thus leading to better parameter estimates.

(2) Comparisons may be made with investment performance results, which are often reviewed quarterly.

In the light of the above considerations, annual intervals are usually used for long-term financial institutions although quarterly intervals are used by Carter (1991). In this paper it is assumed that annual intervals will be used.

(3) What function of each variable should be modelled?

Inflation:

Clarkson (1991) models the year-on-year inflation rate without any modification. Engle (1982), Wilkie (1981, 1984, 1986 and 1987), Carter (1991), FitzHerbert (1992), Pukkila, Ranne and Sarvamaa (1994) and Thomson (1994) model the average force of inflation during each time interval, i.e.:

$$\nabla \ln Q(t) = \ln Q(t) - \ln Q(t-1)$$

where ∇ is the back-difference operator; i.e. $\nabla = 1 - B$; and

$Q(t)$ is the retail price index at time t.

This method ensures that the model is not affected by the choice of time interval. For example, the average force of inflation over two consecutive time intervals is equal to the arithmetic average of the average forces in those time intervals, i.e.

$$\tfrac{1}{2}\{\nabla\ln Q(t)+\nabla\ln Q(t-1)\}=\tfrac{1}{2}\{\ln Q(t)-\ln Q(t-2)\}\ .$$

Because the average force in each time interval is a linear function of other variables, so is the average force over two (or more) consecutive time intervals. This would not be true of average compound rates of inflation. Also, as is pointed out by Hua (1994) (in the context of share price growth), the linear property produces a normal distribution of the output variable if the white-noise term is normally distributed. Furthermore, as is pointed out by FitzHerbert (1992) and Harris (1994), because the forces are additive, the Central Limit Theorem implies that the distribution of aggregated values is asymptotically normal. As pointed out by Clarkson (1991), when inflation rates are low the two approaches are approximately equivalent. However, for a generalized methodology it cannot be assumed that inflation rates will be low.

Equities:

Similarly, in MGWP (1980) and in Wilkie (1984, 1986 and 1987), the average force of growth in dividends during each time interval is modelled. For dividend yields, the logarithm of the dividend yield at the end of each time interval is modelled. Carter (1991), FitzHerbert (1992) and Harris (1994) model the average force of increase of the share price index. This is equivalent to the method of the MGWP and Wilkie, since:

$$\nabla\ln P(t)=\nabla\ln\frac{D(t)}{Y(t)}$$

$$=\nabla\ln D(t)-\ln Y(t)+\ln Y(t-1)$$

where:

$P(t)$ is the share price index at time t;

$D(t)$ is the index of share dividends

$=P(t).Y(t)$; and

$Y(t)$ is the yield on the index at time t.

Because $\nabla\ln D(t)$ and $\ln Y(t)$ are linear functions of other variables, so is $\nabla\ln P(t)$. However, Carter's method does not give a model for dividends.

Another advantage of using the logarithm of the dividend yield is that the model cannot produce negative values of dividends.

For general purposes, the approach of the Wilkie model is well motivated, and is followed by Thomson (1994). In the latter article, similar methods are used for property unit trusts and direct property holdings.

Interest rates:

For money-market instruments, Carter (1991) and Thomson (1994) effectively use the average force of interest during each interval, whereas Ong (1994) uses the discount rate on three-month treasury bills without modification. The choice of instrument depends on the norms for the relevant financial institutions in the country concerned. As explained above, the use of average forces is preferable to the use of annual interest or discount rates.

For government securities, the Wilkie model uses a hybrid expression for the yield on 2 % Consols consisting partly of forces and partly of annual yields. This method, which is discussed further below, is not followed by Carter (1991) or Thomson (1994). Instead, an average annual force of interest is used in those articles, defined as:

$2\ln\left(1+\tfrac{1}{2}y\right)$ for annual intervals; or

$\tfrac{1}{2}\ln\left(1+\tfrac{1}{2}y\right)$ for quarterly intervals

where y is the annual yield to redemption compounded half-yearly.

This method has the advantage of giving values that are consistent and comparable with those used for money-market instruments on the bases used by Carter (1991) and Thomson (1994).

General principles:

In general, the criteria of linearity, consistency and comparability suggest the following principles:

(1) for the modelling of an index used to track growth (such as a retail price index, a share price index or a money-market index), the value modelled should be:

$$\nabla\ln I(t)$$

where $I(t)$ is the value of the index at time t;

(2) for the modelling of a yield used to measure annual cash flow relative to a price or index that has not been modelled on principle 1 (such as the

yield to redemption on an interest-bearing security), the value modelled should be:

$$\ln(1+i)$$

where i is the effective annual yield; and

(3) for the modelling of a yield (such as a dividend yield or rental yield) used to measure one year's cash flow relative to an index that has been modelled on principle 1, the value modelled should be:

$$\ln Y(t)$$

where $Y(t)$ is the yield on the index; provided that, where the yield is implicit in the index (as in the case of a money-market index), principle 1 or 2 (which in such a case are equivalent to each other) should prevail.

It may be found that other functions give a better model than those described above. If so it may be considered appropriate to depart from the above principles, but if a satisfactory model is obtained on those principles it would be better to adopt it than to explore a plethora of unnecessary alternatives. Experimenting for a fit would give less credibility to the results.

In the rest of this article, references to variables being modelled refer to the values of the functions outlined in this section.

(4)(a) For each output variable, which variables (if any) should be used as input variables?

This question relates to the structure of the model. In Wilkie (1984, 1986 and 1987) and Carter (1991), the structure is determined largely on the basis of economic theory. Thomson (1994) makes an attempt to use a methodical approach, based on the data, to derive a structure for the model, and the resulting model is discussed in terms of economic theory. In this article, a generalized method is suggested, based on the data rather than on preconceived economic relationships.

It would be possible to use all variables as input for every output variable and then to eliminate those parameters that are not significantly non-zero, thus eliminating some of the input variables. However, this would involve the simultaneous estimation of a large number of parameters. Furthermore, the elimination of one parameter might significantly change others.

It would also be possible to treat the variables to be modelled, and the white-noise variables, as vectors, thus eliminating the need for transfer functions. This would also involve the simultaneous estimation of numerous parameters, and it would be difficult to interpret the resulting model in terms of economic theory. As pointed out by Claassen and Huber (1992), "Stochastic investment models should ... not be rated solely according to how faithful they are to reality, but also according to how well they represent the user's perception of reality." Conversely, the data may necessitate changes in the user's perception of reality.

Intuitively, the most obvious way to proceed would be to determine the cross-correlation coefficients between the variables being modelled for a range of lags. Where there is significant correlation between a particular variable and subsequent values of another variable, the former could then be used as an input variable for the modelling of the latter. However, because there may be considerable autocorrelation in the values of both variables, the cross-correlation coefficients may be diffused. For this reason a different approach is needed.

Step 1

What is done is to identify and estimate a univariate model for each variable from the observed values. Suppose, for example, that the univariate model for X_t is:

$$X_t = \frac{1-\sum_{i=1}^{p} \theta_{X,i} \cdot B^i}{1-\sum_{i=1}^{q} \phi_{X,i} \cdot B^i} \eta_{X,t} \ .$$

Then the observed values of the input and output variables are transformed by means of the inverse operator:

$$\frac{1-\sum_{i=1}^{p} \phi_{X,i} \cdot B^i}{1-\sum_{i=1}^{q} \theta_{X,i} \cdot B^i} \ .$$

The purpose of this operation, which is known as "pre-whitening", is to transform the autocorrelated input series to an uncorrelated white-noise series. The sample cross-correlation coefficients $\hat{\rho}_{X_t, Y_{t+k}}$ between the pre-whitened input and the correspondingly transformed output are determined.

Values of the chi-squared test statistic $\hat{\chi}^2 = \sum_{k=\tau_0}^{\tau_1} \hat{\rho}^2_{X_t, Y_{t+k}}$ are then determined for suitable intervals $\{\tau_0, \tau_1\}$, e.g.

$\{-5, -1\}$, $\{-5, 0\}$, $\{0, 0\}$, $\{0, 5\}$, $\{1, 5\}$.

The range $\{0, 0\}$ will help to identify those pairs of variables which are cross-correlated at zero lags; if other lags are not significant it is immaterial which variable is selected as input variable and which as output variable . The range $\{0, 5\}$ will help to identify those pairs of variables which are cross-correlated at non-negative lags, while the range $\{1, 5\}$ will help to identify those pairs which are cross-correlated at positive lags. The corresponding negative ranges indicate, where the statistics are significant, that Y should be used as an input variable for the modelling of X.

Step 2

A problem arises with step 1, namely that, unless there are consistent lags between changes in the one variable and changes in the other, it is quite possible that the pre-whitened observed values will not show any correlation, even though the observed values themselves are strongly correlated. This problem is particularly important for models that are being used for long-term applications. Time-series modelling tends to focus on short-term relationships. As a result, the cumulative pressures of growing disparities between two variables may be inadequately reflected.

To check for such effects it is necessary to obtain the cross-correlation coefficients of the observed values of each pair of variables at zero lags. (For the reason indicated above, other lags are not considered.) For those pairs whose cross-correlations are significant but whose pre-whitened values were not found to be significantly cross-correlated in step 1, further consideration is required. Those pairs are considered in this step.

For each of the additional pairs X,Y a modified variable $\tilde{Y}_X$ is defined as:

$$\hat{Y}_{X,t} = \alpha_{X,Y}\left[\frac{1-k_{X,Y}}{1-k_{X,Y}B}\right]X_t + \beta_{X,Y}$$

where:

$k_{X,Y} \in [0, 1)$;

B is the backwards operator; and

$\alpha_{X,Y}$, $\beta_{X,Y}$ and $k_{X,Y}$ are such that

$\sum_t (Y_t - \hat{Y}_t)^2$ is minimized.

$\hat{X}_Y$ is similarly defined.

In calculating these parameters, the operator in the numerator of the expression is not applied as a premultiplier, as would normally be done in time-series modelling. Instead, for each trial value of k, the transformed values $\left[\frac{1-k_{X,Y}}{1-k_{X,Y}B}\right]X_t$ are calculated.

This approach was used by Wilkie (1984, 1986 and 1987) and Thomson (1994) to represent the lagged effect of inflation, with unit gain. Neither article considered the possibility of using this approach for other input variables.

Step 3

Step 1 is now repeated, using the modified variables derived from step 2 in lieu of the corresponding variables initially considered. This gives a preliminary structure for the model. In the process of estimating parameters it may be found that the transfer functions for some of the input variables yield no significantly non-zero parameters. In that case such input variables may be eliminated for the modelling of the output variables concerned.

As mentioned above, the Wilkie model uses a hybrid expression for the yield on 2 % Consols consisting partly of forces and partly of annual yields. The expression is a sum of two terms. The first term is a linear function of sequential values of the force of inflation representing the lagged effect of inflation with unit gain, as described above. The second is intended to represent the "real part" of the Consols yield. Thus the annual yield is effectively expressed as the sum of a force and another quantity. Whether the latter quantity is intended to be a force or a rate is not clear, but either way the model is inconsistent with the suggested general principle (2) under question (3) above.

4(b) to (d) Drawbacks in the application of standard time-series modelling procedures

Shock effects

The question whether outliers (such as the 1987 crash) should be ignored for the purposes of modelling, or whether shock effects should be allowed for is discussed by Carter (1991), Geoghegan et al (1992) and Hua (1994).

Carter (1991) arbitrarily takes outliers as observations outside twice the standard deviation of the adjusted series and makes an arbitrary adjustment to the model to allow for share market crashes about once per decade.

Hua (1994), who uses intervals of one month, makes the monthly return on share prices containing the 1987 crash equal to the estimated mean. This is justified on the grounds that:

(1) there was only one major crash in the data, which meant that it could not be reliably modelled;

(2) the technique used to model crashes has to be different from the model of the variable under normal conditions, since the event is very unusual; and

(3) the exclusion of the month of the crash reduces the skewness of the distribution of the variable.

However, it begs the question whether some of the returns during periods preceding and following the crash were not excessive.

Clarkson (1991) includes random shock terms in a model of inflation rates, which are similar to those used by Carter (1991).

As Geoghegan et al (1992) point out, "mixture models" (such as those of Carter and Clarkson) present a number of problems: "For example, it would be difficult to estimate the appropriate period between shocks, and their appropriate magnitude, form the sparse data available." They suggest that considerations of the incorporation of random shock effects are of more concern for short-term models, and perhaps medium-term models used for extreme values.

Credibility of high-order lags

Another problem relates to the credibility of the model. In a report to the MGWP (MGWP (1980)), E.J. Godolphin discusses a model for the De Zoete Equity Index. One of the models he considers, based on conventional time-series modelling, is an ARIMA(0,1,2) model. However, he notes that an ARIMA(7,1,0) model gives a reduced sample variance of the residuals and proposes the adoption of that model.

Of course, it may be expected that, the larger the number of parameters used, the lower will be the sample variance of the residuals. For this reason a number of order determination criteria are in use (e.g. Schwarz (1978)). Godolphin does not seem to have considered such criteria. The MGWP rejected Godolphin's proposal because monthly prices were needed (which does not seem to have been in Godolphin's brief) and because, "despite the statistical evidence, it was felt difficult to convince people that prices up to seven years earlier had an effect on today's prices."

In the context of dividend yields in the United Kingdom the MGWP (1980) found that although one might conclude on the basis of conventional time-series analysis that a series should be differenced, the resulting variance of long-term forecasts cast doubt on the validity of that conclusion.

As stated by Thomson (1994), for the purpose of analysing the relationships between the variables, short-term lags must be considered; it is not reasonable to presuppose that the value of a variable in a particular year will influence the value of another variable many years later. In the cases considered by the MGWP, the differenced models performed better over short time-intervals. For that reason, for the purposes of developing the structure of the model, differences are taken wherever indicated by the criteria of conventional time-series modelling for the univariate models. However, for the purposes of considering question 4(b), the differencing of output variables should be avoided. The question whether an input variable should be differenced may be decided with reference to an appropriate order determination criterion.

Estimation of means

In general, the parameters may be estimated by ordinary least squares or by maximum likelihood estimation; the latter method, using back-forecasting, is preferable. Programmes exist, such as SAS/ETS, that facilitate these calculations. The author has found, however, that it is preferable to use sample means of the variables (after differencing where necessary) than to include the means in the maximum likelihood estimation; otherwise it may be found that some of the means obtained deviate quite substantially from the sample means, which undermines the credibility of the model.

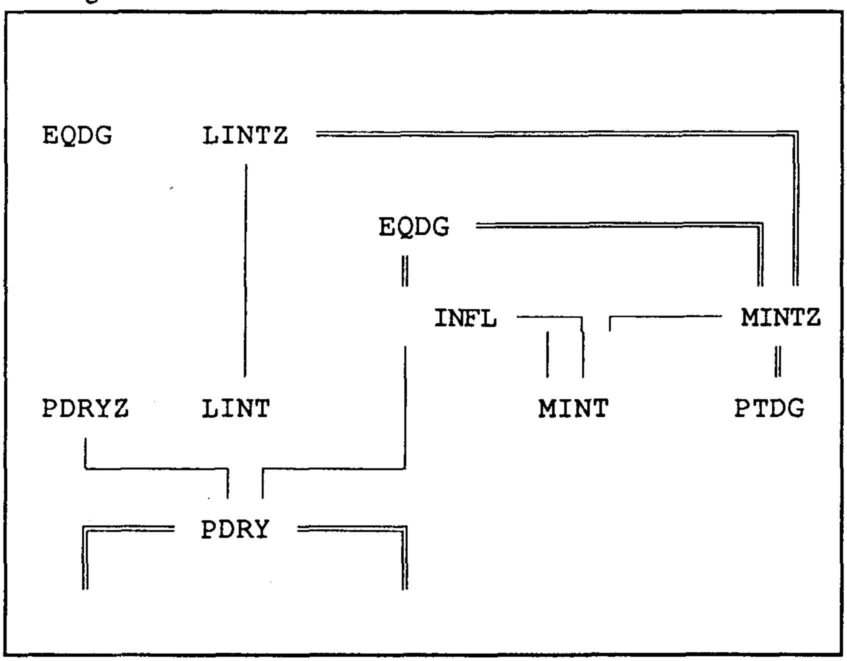
Time-varying parameters

Another matter that needs to be considered is whether time-varying parameters should be used. The testing of ARCH effects and the estimation of the associated parameters is dealt with by Engle (1982). Further generalizations are dealt with by Bollerslev (1986), Clarkson (1991) and Harris (1994). No such tests are made by Wilkie (1984, 1986 and 1987), Carter (1991), Pukkila, Ranne and Sarvamaa (1994) or Thomson (1994). Tests of ARCH effects are made with regard to the Wilkie model by Geoghegan et al. (1992), with positive results. Although no formal test for ARCH effects is made in Hua (1994), such effects are estimated for share price growth in that article. Tests made by Harris (1994) show significant heteroscedasticity of share price growth.

Geoghegan et al (1992) point out that, in the long term, ARCH effects make very little difference, and in the medium term their main effect is on extreme values. The inclusion of ARCH effects therefore depends on the uses to which the model will be put.

Application to South African data

Except as otherwise indicated above, the criteria outlined in this paper are applied to South African data by Thomson (1994). The structure of the resulting model is found in that article to be as shown in the table below.



In that table, double lines represent transfer function and noise models, whilst single lines represent the lagged relationships explained in step 3 of the structure determination procedure, as outlined above. All lines move downwards from input to output variables. The variables are designated as follows with reference to a particular year:

INFL	the average force of inflation during that year;
EQDG	the average force of dividend growth during that year;
EQDY	the logarithm of the dividend yield at the end of that year;
PDRG	the average force of rental growth on direct property during that year;
PDRY	the logarithm of the rental yield on direct property at the end of that year;
PDRYZ	the excess of PDRY over the lagged effect of INFL;
PTDG	the average force of dividend growth on property unit trusts during that year;
PTDY	the logarithm of the dividend yield on property unit trusts at the end of that year;
LINT	the force of interest corresponding to the yield to redemption on long-term interest-bearing securities at the end of that year;
LINTZ	the excess of LINT over the lagged effect of INFL;
MINT	the average force of interest on money-market instruments during that year;
MINTZ	the excess of MINT over the lagged effect of INFL.

Conclusion

The development of a model for the simulation of the cash flows generated by the assets and liabilities of a financial institution is fraught with difficulties, some of which may be insuperable with the theoretical tools currently available, but all of which may be time-consuming. The procedure outlined in this article represents neither an immutable methodology nor a cohesive theory, but it is hoped that it will be of help to practitioners in avoiding some of the pitfalls.

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