FINANCIAL PRICING OF INSURANCE IN THE MULTIPLE LINE INSURANCE COMPANY

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Abstract

This paper uses a contingent claims framework to develop a financial pricing model of insurance that allows for the determination of premium levels by line of business. The model developed explicitly overcomes one of the main shortcomings of previous financial models of insurance : namely, the inability to price insurance in a multiple line insurer subject to default risk. The model implies that it is not necessary to allocate surplus by line ; rather, the price in a given line depends upon the overall risk of the firm and the anticipated inflation rate in the individual line.

The empirical component of the research tests the primary hypothesis derived from the model using a market-value/GAAP database, consisting of publicly traded property-casualty insurers in the United States over the period 1987-1992. The results support the hypothesis : prices vary across firms depending upon overall-firm risk, but within a given firm, prices do not vary by line after adjusting for line-specific inflation. The model also is shown to be more accurate in predicting prices than prior financial pricing models.

Keywords : Surplus allocation, financial pricing, insurance, multiple-line, contingent claims, empirical.

1.1 Introduction

One of the most important developments of the past two decades has been the development of financial pricing models for property-liability insurance and the emergence of the financial actuary. Financial pricing models differ from traditional actuarial models by setting prices appropriate in a market context where there is competition among firms and demand for insurance is sensitive to price. Financial pricing models are consistent with an equilibrium asset pricing model or, minimally, avoid the creation of arbitrage opportunities. For reviews of these models see Cummins (1990, 1992).

A limitation of the existing financial pricing models for property-liability insurance is the implicit or explicit assumption that insurers write only one line of business. Thus, the models fail to provide much guidance for the pricing of insurance in the multiple line firm. Because the vast majority of insurance worldwide is provided by multiple line firms, the single-line focus of existing models is a serious limitation that has been discussed extensively in the actuarial literature (see Kneur (1987) and Derrig (1989)).

The purpose of this paper is to remedy this deficiency in the existing literature by providing a theoretical and empirical analysis of insurance pricing in a multiple line firm. An option pricing approach is adopted in order to facilitate the analysis of pricing in a firm that is subject to default risk. The standard Black-Scholes model is generalized to incorporate more than one class of liabilities and pricing formulae are generated for each liability class. The theoretical predictions of the model are tested using data on an extensive sample of publicly traded U.S. property-liability insurers over the period 1987-1992.

1.2 The Option Pricing Model of the Firm

Option pricing theory has provided important insights into the pricing of property-liability insurance. Option pricing models rely on risk-neutral valuation relationships and/or arbitrage arguments to price the insurance contract. Examples of options models in insurance are Cummins (1988), Doherty and Garven (1986) and Cummins and Danzon (1994). These models represent two significant contributions to the literature on the financial pricing of insurance. First, these models explicitly recognize the probability and expected costs of insurer insolvency. Second, these models incur fewer data estimation problems than competing models such as the Myers-Cohn (1987) or Kraus-Ross (1982) models. Because the options models rely on arbitrage arguments, they eliminate the need to estimate beta coefficients or risk-adjusted discount rates.

The option model of the insurance company draws upon the corporate finance literature, which values corporate liabilities using option pricing theory. In these models, the obligations of insurers to their policyholders are viewed as analogous to risky corporate debt (i.e., debt subject to default risk). In the insurance context, the policyholders "loan" money to the insurer (in the form of premium payments) at time 0 in return for a stream of payments (reimbursement for losses) in the future, just as debtholders loan money to the corporation in return for coupon and principal payments.

Option pricing theory values risky corporate debt as the value of an equivalent default-risk-free bond minus a put option written on the value of the firm. The value at maturity of the riskless bond is L while the value of the put option is Max(0, L-A), where L = the nominal value of the debt and A = the assets of the firm. Thus, at maturity, the debtholders receive L-Max(0,L-A) or Max(L,A), i.e., the debtholders either receive the promised payment (L) or the equity holders default on the obligation and turn over the assets of the firm (A < L) to the debt holders.

Although option models have considerable intuitive appeal, they are not without weaknesses. Although the parameter estimation problems have been reduced, they are not eliminated. There is considerable difficulty in estimating the parameters of the stochastic properties for the liability and assets. In addition, whenever authors have attempted to extend these models into the multiple line case, they have been unable to resolve the problem of allocating equity across lines of insurance.

1.3 Multiple-line insurers

One of the first papers to help solve the problem of how to set premiums for different divisions of the insurance company was by Allen (1993). Allen's unique approach was to question the particular structure that characterizes the typical insurance contract. He begins his discussion by making a number of strong assumptions and establishing a benchmark contract. Then, by relaxing the assumptions, he is able to justify the structure of the standard insurance contract and solve for the premium level in any line of business that would prevail in a competitive market.

The primary insight of the Allen paper is that the surplus account of the insurance company is there to support all of the lines the insurer writes and therefore does not need to be allocated to an individual line of business in order to determine the price of insurance. Allen's paper is important because it explicitly recognizes multiple lines of business. Its principal limitation is the failure to incorporate the risk of insurer insolvency.

The theoretical development in the present paper combines the option modelling approach with the insights that can be drawn from the Allen model. Our model allows for the determination of premium levels by line of business while also recognizing the possibility of insurer ruin or bankruptcy.

Section 2 begins where the Allen model ends. The analysis begins by developing a theoretical model which determines the competitive price of insurance in a mono-line insurance company. The analysis continues by relaxing the assumptions used to arrive at this contract and the end result is a model for which premium levels by line of business, the surplus needed to support those premiums, and the competitive return on the insurance company's equity can all be determined. Section 3 first develops some testable hypothesis directly from the model. This is followed by a discussion of the data collected and the results of the tests. Section 4 investigates the ability of the model to predict actual premium levels by line of business for a number of publicly traded property-casualty companies. The paper concludes with a summary and some final commnets.

2.1 Premium Calculations in the Mono-Line Insurer when Equityholders have Unlimted Liability

Assume that financial markets are complete and informationally efficient. Further assume that a large group of individuals are subject to the possibility of suffering a loss. An insurance company, owned by equityholders, is willing to insure the losses of this group of individuals for a premium P. Assume that equityholders have unlimited liability, i.e., equityholders are liable to pay all losses that arise.

At time 0, premiums, P, are collected by the insurance company and surplus, G, is contributed by the equityholders. For the time being, assume that the premiums and the surplus are invested and held in separate accounts. Assume that the premium and surplus accounts evolve over time according to geometric Brownian motion. Define P and G to be the starting values with starting values of P and G, respectively.

$$\begin{aligned} dP &= \alpha_P P\,dt + \sigma_P P\,dz_P \\ dG &= \alpha_G G\,dt + \sigma_G G\,dz_G \end{aligned} \tag{1}$$

The market value of the premium and surplus account at any time τ is equal to $P(\tau)$ and $G(\tau)$, respectively. τ is defined as the amount of time before the end of the time period. For example, at time 0, τ is equal to 1. At time 1, the insurer agrees to pay the losses incurred by policyholders. Define $L(\tau)$ to be the market value of the firm's liabilities at time τ. Firm liabilities are also assumed to evolve according to geometric Brownian motion, with starting value L.

$$dL = \mu L\,dt + \sigma_L L\,dz_L \tag{2}$$

Equityholders committ surplus at time 0 to act as a cushion against unfavorable outcomes. This is analogous to the margin requirements that brokerage houses demand from investors when they take positions in futures or forward contracts (see Pitts and Fabozzi, 1991). Margin is considered "good faith" money. It is a demonstration on the part of the investor that he will satisfy the obligations of the futures contract no matter what state of the world is revealed. Insurance company equityholders are synonymous with the holder of the futures contract. Surplus is demanded by the policyholders as good faith money to demonstrate that equityholders will satisfy the obligations of the insurance contract. The insurance company is somewhat like the brokerage house:[1] it invests the surplus (or margin) in interest or dividend paying assets. Any losses not covered by the premiums and the investment income they earn will be made up by the equityholders from the surplus account held by the insurance company. Under the unlimited liability assumption, losses in excess of the insurer's assets will be funded by additional contributions from the equityholders.

At the end of the time period, i.e. $\tau=0$, there will be three relevant cash flows. The premium will be determined by the value of these cash flows at time 0. The first cash flow is the loss payment from the insurer to the policyholders. The second can be modeled as a call option, $C(P,L,\tau)$.[2] This option is held by equityholders and gives them the right to any residual value which may be left in the premium account after all liability obligations have been met. The third cash flow can be modeled as put option $B(P,L,\tau)$, held by the policyholders, which entitles them to additional money from the equityholders if the premium and investment income is not enough to cover all of the losses.

As pointed out by Allen (1993) the investment strategy adopted by the insurer in investing the premiums and equity at time 0 is theoretically indeterminate. To see this, consider the case where the insurance company invests the premiums it collects in a risky asset portfolio with an expected return of $\alpha_P=r^*$. The premium paid at time 0 would be

$$P = Le^{-(r^*-r_L)\tau} - C(P,L,\tau) + B(P,L,\tau). \tag{3}$$

The premium P is equal to the discounted expected value of the liability reduced by the "payment" the equityholders implicitly make to the policyholders for the value of the call option, plus the value of the put option made by the policyholders to compensate the equityholders for guaranteeing the residual risk.

Note that depending upon the investment strategy undertaken by the firm, different policyholders will be charged different prices for the same contract. Does this mean that policyholders are better off if they pay a lower price? No. As Allen (1993) showed, policyholders will offset the investment strategy of the insurer by adjusting their own optimal portfolios. This result is just an application of the well known Modigliani-Miller (1958) theorem to insurance. The M-M theorem states that in a world with perfect capital markets and no taxes, the capital structure a firm employs does not affect the value of the firm.

[1]In actuality, the brokerage house is an independent agent acting on behalf of the investor. The equityholders in this model are assumed to fully control the insurance company, i.e., there are no managers at the insurance company acting on behalf of the equityholders.

[2] The details of how the option values are determined is presented in the Technical Appendix.

The value of the firm at any time is equal to the value of the assets of the firm and by definition, equal to the value of the equityholders claim, EH(τ), plus the policyholder's claim, PH(τ). At time 0, the market value of the firm's assets is P, given by equation (3). The value of the policyholder's claim at time τ, i.e., (1-t), is equal to

$$PH(\tau) = A(\tau) - EH(\tau) = P(\tau) - [C(P,L,\tau) - B(P,L,\tau)]. \quad (4)$$

Proposition 1: With unlimited liability, the value of the policyholder's claim is always equal to the discounted value of the expected liability, discounted at $(r_f - r_L)$.

Proof

Recall the well known put-call parity relationship. The relationship states that the value of a call option with exercise price K written on an underlying stock is always equal to the value of the put option with exercise price K plus the value of the stock minus the discounted value of the exercise price discounted at the risk-free rate. Using the insurance terminology developed in the previous section this would require that the call option that the equityholders of the firm own minus the put option that the policyholders own is always equal to the premium collected minus the discounted expected value of the liability discounted at the risk-free rate minus the liability inflation parameter r_L, i.e., $C(P,L,\tau) - B(P,L,\tau) = P(\tau) - Le^{-(r_f - r_L)\tau}$.[3] Using this relationship

$$PH(\tau) = P(\tau) - [C(P,L,\tau) - B(P,L,\tau)] \quad (5)$$

$$= P(\tau) - [P(\tau) - Le^{-(r_f - r_L)\tau}] \quad (6)$$

$$= Le^{-(r_f - r_L)\tau}. \quad (7)$$

QED

Intuitively, Proposition 1 holds because of the unlimited liability assumption. No matter what state of the world occurs, policyholders will always receive the full value of their claim.

Define the risk-adjusted discount rate to be the discount rate which sets the present value of the liability equal to the policyholders claim on the firm divided by the expected liability payment (see Merton (1974)):

$$Le^{-r_a\tau} = PH(\tau). \quad (8)$$

[3] The most common version of the put-call parity relationship discounts the exercise price by just the risk-free rate. The discount rate used in this version of the put call parity relationship is adjusted by the liability drift parameter due to the uncertain exercise price. See Fisher (1978).

Therefore, using equation (7) the risk-adjusted discount rate when equityholders have unlimited liability is just the risk-free rate minus the inflation component of the liability drift term, r_L, regardless of the investment strategy chosen by the insurance company.

> *Proposition 2: With unlimited liability, when the investment strategy undertaken by the firm is to invest the premiums in the risk-free asset, the value of the call option that the equityholders hold and the value of the put option that the policyholders hold are always equal.*

Proof

In the proof from Proposition 1, it was shown that $C(P,L,\tau)-B(P,L,\tau)=P(\tau)-Le^{-(r_f-r_L)\tau}$. Suppose the investment strategy undertaken by the firm is to invest all premiums into the risk-free asset. Then by equation (3)

$$P(\tau) = Le^{-(r_f-r_L)\tau} - C(P,L,\tau) + B(P,L,\tau) \tag{9}$$

$$P(\tau) - Le^{-(r_f-r_L)\tau} = -C(P,L,\tau) + B(P,L,\tau) \tag{10}$$

Now using the put-call parity relationship, (10) becomes

$$C(P,L,\tau) - B(P,L,\tau) = -C(P,L,\tau) + B(P,L,\tau) \tag{11}$$

$$2C(P,L,\tau) - 2B(P,L,\tau) = 0 \Rightarrow C(P,L,\tau) = B(P,L,\tau). \tag{12}$$

QED

Intuitively this makes sense. Think of the stock option analogy. When the price of the stock is equal to the discounted value of the exercise price both the call and the put option are out-of-the money. Because of the symmetry of the underlying returns distribution, both the call option and the put option have the same value. The same is true in the insurance context. When the discounted expected value of the liability claim is discounted by the risk-free rate minus the liability inflation parameter, the call option and the put option are equal.

The expected return that shareholders receive is a combination of the return they anticipate to earn on the surplus held by the firm and on the compensation they receive for bearing the residual risk. For the purposes of this paper, assume the investment strategy for the insurance company to adopt on behalf of the equityholders is invest the surplus G in the risk-free asset. Using this assumption, the expected return on equity at time 0 would be

$$r_E = \frac{(1+r_f)G + E[MAX(P-L,0)] - E[MAX(L-P,0)]}{G + C(P,L,1) - B(P,L,1)} - 1. \tag{13}$$

2.2 Premium Calculations with Limited Liability in the Mono-Line Insurer

Now assume that equityholders have limited liability, i.e., equityholders are only liable to pay losses until the assets of the company have been depleted. In the event there are remaining losses to be paid, the equityholders declare bankruptcy and walk away, turning the assets of the firm over to the policyholders. In a competitive market with complete information, policyholders will take this limited liability position into account, when deciding how much they are willing to pay for the insurance contract.

At time 1, i.e. $\tau=0$, there are four relevant cash flows. The premium will be determined by the value of these cash flows at time 0. The first cash flow is the loss payment from the insurer to the policyholders. The second can be modeled as a call option, $C(P,L,\tau)$. This option is held by equityholders and gives them the right to any residual value which may be left in the premium account after all liability obligations have been met. The third cash flow can be modeled as put option $B(P,L,\tau)$, held by the policyholders, which entitles them to additional money from the equityholders if the premium and investment income is not enough to cover all of the losses. Finally, there is a second put option held by the equityholders. This second put option is called the insolvency put, $I(P+G,L,\tau) = I(A,L,\tau)$. The insolvency put gives the equityholders the right to default on any losses remaining after the entire firm has been liquidated.

The premium paid at time 0 is a function of the value of the cash flows that are generated as a result of the transaction.

$$P = Le^{-(r^*-r_L)\tau} - C(P,L,\tau) + B(P,L,\tau) - I(P+G,L,\tau). \tag{14}$$

where

r^* = expected return on the invested premiums

r_L = liability growth rate.

The policyholder's claim with τ time periods remaining until expiration is equal to the value of the firm minus the value of the equityholders' claim on the firm.

$$PH(\tau) = A(\tau) - EH(\tau) \tag{15}$$

$$= P(\tau) - [C(P,L,\tau) - B(P,L,\tau)] - I(A,L,\tau) \tag{16}$$

Using the put-call parity relationship, equation (16) becomes

$$= P(\tau) - [P(\tau) - Le^{-(r_f-r_L)\tau}] - I(A,L,\tau) \tag{17}$$

$$= Le^{-(r_f-r_L)\tau} - I(A,L,\tau). \tag{18}$$

Equation (18) says the value of the policyholder's claim on the firm is equal to the risk-free claim minus the value of the insolvency put option. The result is similar to the standard result from the risky debt pricing literature (see Merton (1974)).

The risk-adjusted discount rate, r_d, is the discount rate which sets

$$L e^{-r_d \tau} = PH(\tau). \tag{19}$$

Taking the logs of both sides and solving for r_d you get

$$r_d = \frac{-1}{\tau} \ln \frac{PH(\tau)}{L} > (r_f - r_L). \tag{20}$$

The expected return on equity is equal to the equityholders expected payout divided by the value of the equityholder's claim on the firm at time 0. Assuming the equityholder's guaranteeing surplus is invested in the risk-free asset, the expected return on equity is:

$$r_E = \frac{(1+r_f)G + E[MAX(P-L,0)] - E[MAX(L-P,0)] + E[MAX(L-A,0)]}{G + C(P,L,1) - B(P,L,1) + I(A,L,1)} - 1 \tag{21}$$

The expected return on equity depends on the amount of surplus committed to the firm and the amount of residual risk that the surplus is subject to.

2. 3 Premium Calculations in the Multiple-Line Insurance Company with Unlimited Liability

Now consider what happens when the insurance company consists of two divisions[4] and equityholders again have unlimited liability. As before, in return for premiums paid at time 0 by the policyholders, the equityholders agree to pay all losses which arise. Each division manager is allowed to adopt a separate investment strategy and/or underwrite liabilities which evolve according to different stochastic processes. Assume that equityholders input surplus at time 0 of G.[5] At the end of the time period, if value of division i's asset portfolio is greater than L_i, the policyholders are paid and the rest of the money is paid out to the equityholders as dividends. The value of this claim can be modeled as a call option, i.e. $C_i(P_i,L_i,0)$ for i=1,2. Likewise, if the value of division i's asset portfolio is less than L_i, than the equityholders must make up the difference. The value of this claim can be thought of as a put option, $B_i(P_i,L_i,0)$.

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[4]The case for more than two divisions is straight forward.

[5]Even with unlimited liability, equityholders may be required by policyholders to put up surplus at time 0 to reduce the costs of enforcing the terms of the contract in the event the premiums and investment income do not cover all of the realized losses. This will be discussed in more detail later.

The premium charged in each line of business will be determined in a way similar to the equation used to determine the premium for the mono-line insurer. Suppose the optimal investment strategy for policyholders in division i is for the company to invest the premiums in risky assets with expected return r^i. The premium that would prevail at time 0, i.e. $\tau = 1$, in division i is

$$P_i = L_i e^{-(r^i - r_{L_i})\tau} - C_i(P_i,L_i,\tau) + B_i(P_i,L_i,\tau). \tag{22}$$

Note that the premiums the policyholders are willing to pay is still independent of the amount of "good faith" margin G the equityholders put up. This is because the equityholders are liable for any losses that arise even if the surplus of the company has been depleted.

The value of the policyholders' claim for class i with τ periods until expiration is equal to the assets dedicated to that line of business minus the value of the equityholders claim on that line of business. The assets dedicated to a particular line of business does not include any equityholder surplus. The only claim that policyholders have on the surplus of the company comes through the put option that it owns.

$$\begin{aligned} PH(\tau)_i &= P(\tau)_i - EH(\tau)_i \\ &= P(\tau)_i - [C_i(P_i,L_i,\tau) - B_i(P_i,L_i,\tau)]. \end{aligned} \tag{23}$$

Proposition 3: The value of the policyholder's claim for each line of business is always equal to the expected liabilities discounted at $(r_f - r_{L_i})$ when equityholders have unlimited liability.

Proof

The proof is similar to the one used to prove Proposition 1.

The risk-adjusted discount rate for division i of the firm is

$$r_{d_i} = -\frac{1}{\tau}\ln\left(\frac{PH(\tau)_i}{L_i}\right). \tag{24}$$

Note, just as was the case for the mono-line insurance company, when equityholders have unlimited liability, r_{d_i} is always equal to $(r_f - r_{L_i})$.

The return on equity, r_e, is again a combination of the expected return on surplus held for the equityholders plus the expected payout from the call options minus the expected payout from the put options the policyholders hold. If we continue to assume that the optimal strategy for the insurance company is to invest the surplus G in the risk-free asset, than the expected return is

$$r_e = \frac{(1 + r_f)G + \sum_i E[MAX(P_i - L_i, 0)] - E[MAX(L_i - P_i, 0)]}{G + \sum_i C(P_i, L_i, 1) - B(P_i, L_i, 1)}. \tag{25}$$

2.4 *Premium Calculations in the Multiple Line Insurance Company with Limited Liability*

Now assume the insurance company has two lines of business and the equityholders of the firm are not liable for losses beyond the value of the firm. Policyholders will again take this limited liability position into account when deciding how much they are willing to pay for the insurance contract.

The amount policyholders will pay in premiums depends upon, in part, on the payout they can expect in cases of bankruptcy. Assume in cases where the liabilities of the firm are greater than the assets at time 1, the policyholders will receive the assets of the firm according to an *equal priority rule.* The equal priority rule states that in cases of bankruptcy the assets of the firm will be divided among the policyholders according to the proportion of total liability claims they hold against the firm. Therefore, each class of policyholders will receive w_{L_i} percent of the total assets of the firm, where

$$w_{L_i} = \frac{L_i}{L} \tag{26}$$

where L is the total liabilities of the firm. This equal priority rule is consistent with other academic literature that has modeled insurance insolvencies (see Cummins and Danzon (1994)). It is also consistent with the way insurance bankruptcies are handled in practice (Kimball and Denenberg (1969)).

The informationally efficient competitive premium that policyholders are willing to pay now is equal to:

$$P_i = L_i e^{-(r' - r_{L_i})\tau} - C_i(P_i, L_i, \tau) + B_i(P_i, L_i, \tau) - w_{L_i} I(A, L, \tau). \tag{27}$$

The premium for policyholder class i is equal to the present value of the liabilities discounted at the expected return on premium account the for the policyholders of the line of business, adjusted by the options. The premium is reduced by the value of the divisional call option help by equityholder, increased by the value of the divisional put option held by policyholders, and reduced by w_{L_i} percent of the value of the insolvency put option held by equityholders hold due to their limited liability position. Note that the values of the divisional options are completely determined by the division of the firm. Only the insolvency put option is a function of the whole value of the firm.

The risk-adjusted discount rate, r_{d_i}, for division i of the firm is the rate that sets the discounted value of the policyholder's claim for division i equal to the promised payoff at time 1. The value of the policyholder's claim for division i when equityholders have limited liability is equal to

$$
\begin{aligned}
PH(\tau)_i &= P(\tau)_i - EH_i \\
&= P(\tau)_i - [C_i(P_i,L_i,\tau) - B_i(P_i,L_i,\tau) + w_{L_i} I(A,L,\tau)] \qquad (28) \\
&= L_i e^{-(r_f - r_{L_i})\tau} - w_{L_i} I(A,L,\tau).
\end{aligned}
$$

The expected return on equity, r_e, is similar to the formula (25) except it is adjusted to account for the value of the insolvency put.

$$
r_e = \frac{(1+r_f)G + \sum_i \{E[MAX(P_i - L_i, 0)] - E[MAX(L_i - P_i, 0)]\} + E[MAX(L - A, 0)]}{G + \sum_i \{C_i(P_i,L_i,1) - B(P_i,L_i,1)\} + I(A,L,1)}. \qquad (29)
$$

One final point should be made in this section. Various authors have argued that with limited liability, the price of insurance for a particular line of business is a function of the amount of surplus which is allocated to that line of business. For example, Doherty and Garven (1986), Myers and Cohn (1987), and Kneur (1987) argue that surplus must be allocated to various lines of business in order to determine the fair value of insurance for a particular line of business. In this paper, we argue otherwise. What is important in determining fair insurance premiums is the residual risks that policyholders face. The allocation of surplus to a particular line of business implies that a different line of business does not have access to the surplus which is supporting other lines. This is not the case. Surplus is used by the company as a cushion against unfavorable realized states of the world and it is available to any division manager that requires it. Therefore, policyholders should focus on the payouts they can expect from the company in all possible outcomes of the world. It is the total amount of equity that the company has and the payouts they can expect from the company which will determine the fair value of insurance.

3.1 Empirical Tests: Introduction

The implications of the model presented in the previous section were investigated by conducting two empirical tests. The first test examines the price of insurance both across insurers and for different lines of insurance within insurers. The second analysis examines the ability of the model to predict actual aggregate premium levels by line of business.

3.2 Insurance Price Differences Across and Within Insurance Companies

The first part of this inquiry investigates the price of insurance for a given line of business across different insurers. To study the price of insurance, researchers often use the premium ratio (see Berger, Cummins and Tennsyson (1992)). This ratio can be viewed as the unit price of insurance, i.e., the price of insurance per dollar of losses expected to be paid. Using the notation developed in Chapter 2, the premium ratio for line of business i in company j, PR_{ij}, equals

$$PR_{ij} = \frac{P_{ij}}{E(L_{ij})} = \frac{P_{ij}}{L_{ij}e^{r_{L_i}\tau}} \tag{30}$$

where

P_{ij} = premium paid at time 0 for line i in company j,
L_{ij} = starting value for liability process for line i in company j,
r_{L_i} = instantaneous expected liability growth rate for line i in company j,
τ = time until maturity.

Using the formula for the premiums that would prevail in a competitive and informationally efficient market (equation (27)) equation (30) becomes[6]

$$\frac{P_i}{L_ie^{r_{L_i}\tau}} = \frac{L_ie^{-(r^i-r_{L_i})\tau} - C_i(P_i,L_i,\tau) + B_i(P_i,L_i,\tau) - w_{L_i}I(A,L,\tau)}{L_ie^{r_{L_i}\tau}}. \tag{31}$$

where,

r^i = expected return on the asset portfolio for line of business i,
C_i = divisional call option for line of business i,
B_i = divisional put option for line of business i,
w_{L_i} = liability weight for line of business i,
$I(A,L,\tau)$ = insolvency put for the entire insurance company.

Define σ_x to be volatility of the assets of the firm[7], differentiating equation (30) with respect to σ_x yields

$$\frac{\partial PR_i}{\partial \sigma_x} = -w_{L_i}\frac{\partial I}{\partial \sigma_x} < 0. \tag{32}$$

[6] For clarity, we will drop the j subscript for the time being.

[7] See the Technical Appendix.

Since the value of an option increases as the underlying asset volatility increases,[8] relation (32) is negative. This yields the first proposition.

Hypothesis 1

In a competitive market with perfect information, low risk firms will command a higher price of insurance than high risk firms.

The hypothesis that there is an inverse relationship between firm risk and insurance premiums is not new in the literature. For instance, the hypothesis is consistent with the result Cummins and Danzon (1994) report for all lines. They used the leverage ratio as a proxy for firm risk and found that it was inversely related to the premium ratio. In addition to the insurance literature, there is a vast theoretical and empirical literature investigating the relationship between firm risk and the cost of firm debt.[9]

The second hypothesis concerns the relationship between prices of insurance within the same insurance company. Expanding equation (31) yields

$$\frac{P_l}{L_l e^{r_{L_l}\tau}} = \frac{L_l e^{-(r^l - r_{L_l})\tau}}{L_l e^{r_{L_l}\tau}} - \frac{C_l(P_l,L_l,\tau) - B_l(P_l,L_l,\tau)}{L_l e^{r_{L_l}\tau}} - \frac{w_{L_l} I(A,L,\tau)}{L_l e^{r_{L_l}\tau}} \qquad (33)$$

Now using the Put-Call Parity relationship and the definition of w_{L_l}, equation (33) reduces to

$$\frac{P_l}{L_l e^{r_u\tau}} = e^{-r^l\tau} - \frac{P_l - L_l e^{-(r_f - r_u)\tau}}{L_l e^{r_u\tau}} - \frac{\frac{L_l}{L} I(A,L,\tau)}{L_l e^{r_u\tau}} \qquad (34)$$

$$\frac{P_l}{L_l e^{r_{L_l}\tau}} = \frac{1}{2}\left[e^{-r^l\tau} + e^{-r_f\tau} - \frac{I(A,L,\tau)}{L e^{r_{L_l}\tau}}\right] \qquad (35)$$

Assume that r^i, i.e., the expected return on the investment portfolio for line of business i, is the same for each line of business within the same insurer. With this assumption, it is easy to see from equation (35) that when two lines of business in the same insurance company have identical liability growth rates, their premium ratios should be equal. Furthermore, if it is the case that the insolvency put, $I(A,L,\tau)$, is equal or very close to

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[8] See Cox and Rubinstein (1985).

[9] This literature begins with the seminal theoretical work of Merton (1974) and has produced numerous empirical studies to validate the hypotheses.

zero, then premium ratios of two lines of business within the same insurance company will be equal, even when there are differences between their growth rates and instantaneous risk or standard deviation parameters.[10] This yields the second proposition.

Hypothesis 2

The ratio of any two premium ratios within the same insurance company will always be equal to one assuming the insolvency put is close to zero.

The hypothesis that the premium ratio across lines of business within the same insurance company will always be equal to one is a vastly different prediction than the one that is typically drawn in the insurance literature. Many insurance pricing models predict that the differences between premium ratios across lines of business within the same insurance company are a function not only of the line-specific inflation rate but also of the riskiness of the line of business. This prediction arises in both the actuarial (e.g. Beard, Pentikainen and Pesonen (1984)) and financial (e.g., Myers and Cohn (1987), Derrig (1989)) literature. In general, any surplus allocation which relies on line-specific risk would lead to premium ratios varying by line of business. The model presented in this paper, on the other hand, implies that surplus allocation is unnecessary to determine the competitive price of insurance and therefore should have no impact upon the premium ratio for any line of business. It is the standard deviation, i.e. the riskiness for the *entire* value of the firm that is the relevant risk parameter, not the individual line of businesses standard deviations. It is the interaction of the riskiness of each line of business together with the riskiness of the asset portfolio that determines the risk level of the firm and, therefore, the informationally efficient competitive price of insurance.

3.3 Data and Methodology

To test the hypotheses developed in section 3.2, a database of publicly traded property/liability insurers over the time period 1987-1992 was constructed. Selection criteria for inclusion in the study were that the firm be either a property/liability insurance company, or a multi-line insurer with at least 25% of its business in property/liability insurance. In addition, firms which became insolvent or experienced severe financial difficulties were eliminated. Severe financial difficulty was defined as any firm for which direct regulatory intervention was required for the firm to remain solvent.

The risk measure employed in this paper was the annualized standard deviation of the firm's equity returns. The returns were collected from the NYSE/AMEX and

[10] As discussed in section 3.3, the sample of companies included in this study consist of only those firms which did not have any 'significant solvency problems' during the sample period 1988 through 1992. An example of a 'significant solvency problem' would be direct regulatory intervention into the ongoing operations of the firm.

NASDAQ CRSP tapes. The annualized standard deviation of the firm's equity is based on the forty weekly equity returns before the end of the year. Weekly, as opposed to daily returns, were used because a number of the firms included in this sample are small insurers whose stocks trade infrequently. Nonsynchronous trading can result in substantially biased estimates of assets returns.[11] Using weekly instead of daily returns can help to minimize this nontrading effect. Forty weekly returns were used as a compromise between two competing objectives. Because equity volatilities have been shown not to be constant over time, the most recent serves as the best guide to how equity volatility will evolve in the near future. However, the larger the sample of calculated returns, the more powerful the test statistic will be. Therefore, a good balance between these two competing objectives was thought to be the forty most recent weekly equity returns.

Two different definitions of the dependent variable were used in this study: the premium ratio and the economic premium ratio. The model developed in Section 2 suggests the premium ratio is the correct variable to be investigated. However, the model is a one period model which assumes all losses are paid at the end of the time period. Many long-tailed lines of business still have claims remaining to be paid five or ten years after the coverage period has expired. Therefore, to control for the loss payout patterns of various lines of business, the economic premium ratio was also used. The economic premium ratio discounts the expected loss payments to the end of the policy year at the risk-free rate. Using the economic premium ratios allows for direct comparisons between long and short-tailed lines of business.

The data to estimate the ratios came from the *Insurance Expense Exhibit* (IEE) which is available from the A.M. Best Company. The years included in this study were 1988 through 1992. The definition of the economic premium ratio is

$$EPR_{ij} = \frac{NPW_{ij} - DIV_{ij}}{(NLI_{ij} + LAE_{ij}) \times PVF_{ij}} \tag{36}$$

and the premium ratio is

$$PR_{ij} = \frac{NPW_{ij} - DIV_{ij}}{NLI_{ij} + LAE_{ij}} \tag{37}$$

where

NPW_{ij} - net premiums written for line i, company j

DIV_{ij} - policyholder dividends paid for line i, company j

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[11]See Lo and MacKinlay (1990) for an analysis of the effect of nonsynchronous trading on the time series properties of assets returns.

NLI_{ij} - net losses incurred for line i, company j
LAE_{ij} - net loss adjustment expenses incurred for line i, company j
PVF_{ij} - present value factor for line i, company j.

To control for differences in underwriting expenses across lines of business versions of the economic premium ratio and the premium ratio were also calculated which control for underwriting expenses by subtracting the net underwriting expenses from the numerators of equations (36) and (37). This step is potentially important because expense ratios vary substantially across lines of insurance. In calculating the ratios, lines of business were grouped into short- and long-tail categories. Lines of business which generally pay 90% of claims within three years were considered short-tail lines, while lines that take longer to close are considered long-tailed lines (line definitions are provided in Appendix 2). The present value factors used to discount the loss cash flows are based upon total reserve development factors estimated from Schedules O and P based upon total industry data. This data is available in *Best's Aggregates and Averages* (1986-1993). The discount rates up through 1989 are the U.S treasury yield curves reported by Coleman, Fisher, and Ibottson (1989). For the years 1990 through 1992, yield curves were calculated from spot rates of U.S. Government STRIPS.[12]

The loss payout patterns for all years used in this study are not available from A.M. Best's.[13] However, the payout patterns for individual lines of business tend to be relatively stable. Accordingly, for missing years, the average of the years which were available was used. This approximation did not seem to have a large effect upon the results.

In addition to the missing data problem, Best's aggregates a number of individual lines of business into a single line. For instance, Aircraft, Boiler and Machinery, and Ocean Marine insurance are combined and reported as Special Liability. Thus, the same present value factor calculated on the aggregate data is applied to each component which constitutes the composite line. This approximation also did not seem to affect the results greatly.

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[12] STRIPS (Separate Trading of Registered Interest and Principal Securities) are government bonds whose coupon payments have been "stripped" away leaving only the principal payment due at the time of maturity, i.e., zero coupon bonds. Thus the yield curve can be observed directly.

[13] For example, data was reported for international insurance in 1988 and 1990, but not 1989.

Observations with premium or economic premium ratios less than 0 or greater than 5 were eliminated. Also, observations for which there was only partial data available were eliminated. This left a sample of 77 companies over the time period 1988-1992.

Summary statistics for the companies in the sample are presented in Table 3.1. A list of companies used in this study can be found in Appendix 1. As expected, the average economic premium ratio for both long and short tail lines is higher than the corresponding non-discounted premium ratio. Also, the premium ratios which eliminate the underwriting expenses are smaller than its corresponding ratio. The average annualized standard deviation of equity is .33249 which is consistent with the results reported for other financial intermediaries.

3.4 Test Methodology

To test the hypothesis that premiums are inversely related to the riskiness of the firm, two versions of the following equation were estimated.

$$PR_{ij} = \alpha_{ij} + B_{ij}Sigma\,E_{ij} + \theta_{ij}Year\ Dummy_{ij} + \gamma_{ij}Log(Assets)_{ij} + \pi_{ij}Notrades_{ij}. \quad (38)$$

The natural logarithm of the market value of the assets of the firm was included to control for any effect firm size may have in explaining price difference. The variable Notrades is equal to the number of days over the course of the year for which there was no trading in the stock of the firm, or for which the price listed in CRSP is not a closing price. This variable was included to control for bias induced by the nonsynchronous trading problem discussed earlier.

Two versions of the regressions were estimated, using a pooled cross-section, time-series approach: a fixed effects regression and a random effects regression. The fixed effects model uses year and company dummy variables to control for any specific time and firm effects. The random effects model specifies the error term to allow for company, year, and observation-specific components.[14] Two versions of the equation were estimated because it is not clear *a priori* which approach is more appropriate for this particular data base.

To test the second hypothesis, that premium ratios across lines of business within the same insurer are equal, the ratio of the short-tailed over the long-tailed economic premium ratio controlling for expenses was calculated for each company. The first test then estimated was a simple t-statistic testing the null hypothesis that this ratio was equal to 1. A second slightly more sophisticated test was to estimate a fixed-effects regression which controls for time and company variation using dummy variables. A t-test was then calculated to test the null hypothesis that the intercept was equal to one. Finally, a

[14] See Greene (1990), chapter 16, for a discussion of the random and fixed effects methodologies.

random-effects regression was estimated and a t-test was again calculated to test the null hypothesis that the intercept was equal to one.

3.5 Test Results: Price Differences Across Insurers

The results of the regression based upon equation (38) are reported in Table 3.2.[15] Because the conclusions based on the fixed and random effects models are similar, only the random effects models are shown. The fixed effect results are available from the authors on request.

In Table 3.2, SIGMAE is negative and significant at either the 1% or 5% level for all long-tailed equations. SIGMAE is never significant in any of the short-tailed runs. The results based on the fixed-effects regressions are similar -- SIGMAE is always significant for the long-tail lines. In these regressions, SIGMAE has a positive coefficient for the short-tail lines but is insignificant in the short-tail regressions. In general, therefore, the results suggest that firm risk is inversely related to the premium ratios for long-tailed lines and has no measurable affect on short-tailed lines.

One possible explanation for the finding that firm risk is of more concern to policyholders in long-tailed lines of business and of little concern to policyholders in short-tail lines is due to the greater uncertainty of the long-term claims paying ability of the firm. The longer the payout tail, the more uncertainty there is about the solvency of the firm and therefore, the more policyholders will penalize the firm. Consider the analogous result in the risky-debt literature. Merton (1974) showed in an options framework that for firms which were not already bankrupt (in the risk-neutral sense), the spread between the yield on their risky debt and the risk-free rate becomes larger the longer the term to maturity.[16] This result has been verified empirically for corporate bonds by Litterman and Iben (1991).

The variable NOTRADES, which was included to control for any bias induced by non-trading of some firm's stock included in the sample is never significant in either model. Therefore, the nonsynchronousity of the firm's stock does not independently explain any price variation across companies.

The variable LMVA, the natural logarithm of the market value of the firm's assets, was included to capture any effect firm size may have in explaining variation in the dependent variable. It is negative and highly significant for all long-tail equations in each

[15] Note, the number of companies used in the fixed effects model was 77. The random effects model requires full panels of data for each company in order to run. Eliminating those companies with missing data left 58.

[16] Actually, Merton found that after a certain point, the yield spread actually began to decline for very long maturities. However, the yield spread never decreased enough to become smaller than the yield spread for very short maturities.

model. In short-tailed lines the effect is not as pronounced. It is only significant in half of them and only at the 10% level. This suggests that larger firms are able to charge lower prices in long-tailed lines. In short-tailed lines, smaller firms are better able to compete with the larger companies. A possible explanation is that the costs to policyholders of monitoring the insurer may be higher in larger firms because of the complexity and scope of operations of larger insurers. Therefore, policyholders may "charge" the insurer by way of lower premiums for the higher monitoring costs.

3.6 Test Results: Price Differences Across Lines Within Insurers

The results of the second hypothesis tests, that the economic premium ratios within the same insurer are equal, are presented in Table 3.4. The top panel of the table tests the ratio of the economic premium ratio for all short-tail lines to the ratio for all long-tail lines. Recall that the null hypothesis is that this ratio should equal 1.0. In each of the three tests in the top panel, short-tail lines of business had a significantly higher economic premium ratios than long-tail lines at either 1% or 5% level. Therefore, this test yields little support for the hypothesis.

One possible reason for the lack of support for the hypothesis is the existence of restrictive rate regulation, which primarily affects the two most important long-tail lines, workers' compensation and autmobile liability.[17] Thus, to investigate the hypothesis further, workers' compensation and automobile liability insurance were removed from consideration and the tests were rerun. The results are shown in the lower panel of Table 3.3. While a simple t-test again leads to rejection of the null hypothesis that the short and long-tail economic premium ratios are the same within each insurer, after controlling for the cross-sectional and times series variation in the fixed effects and random effects regressions, one cannot reject the null hypothesis of equal economic premium ratios within the same insurer. Thus, after controlling for regulation and company and time effects, we find evidence supporting the hypothesis.

3.7 Accuracy Test

In order to obtain accurate estimates of aggregate premium levels by line, a number of additional variables need to be estimated. Consider the premium equation once again

$$P_t = L_t e^{-(r^t - r_{L_t})} - C_t(P_t,L_t,\tau) + B_t(P_t,L_t,\tau) - w_{L_t} I(A,L,\tau). \tag{39}$$

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[17]Both lines of business are characterized by large involuntary markets in many states and there are often threats of private insurers abandoning the market due to inadequate rates. Harrington (1987) and Grabowski, Viscusi and Evans (1989) found that rate regulation held premium ratios for automobile liability insurance below competitive levels during the mid-1980's. Evidence on the effects of regulation on workers' compensation insurance prices is provided by Carroll (1993).

One way to estimate premium volumes by line is to determine the value of each option in equation (39). This would require getting good estimates of the underlying volatility of each line and also a good estimate of the overall volatility of the firm. However, there is an easier way. Recalling the relationship between the divisional call and the divisional put option, equation (39) becomes

$$P_i = \frac{1}{2} [L_i e^{-(r^i - r_{L_i})\tau} + L_i e^{-(r_f - r_{L_i})\tau} - w_{L_i} I(A, L, \tau)] \tag{40}$$

Now, all we need to predict premiums by line are good estimates of the insolvency put, the liability weights, w_{L_i}, the starting value of the liability process, L_i, the amount of guaranteeing surplus put up by the equityholders of the firm, G, and the risk-free rate r_f and the expected return on assets, r^i. Estimating the underlying volatility of each line of business or the divisional put or call is not necessary.

The initial surplus, G, used in the test was the market capitalization of the firm at the end of the previous year. This implicitly assumes that any outstanding claims can be paid from the loss reserves held by the insurer. The total assets used to determine the value of the insolvency put are equal to the starting value of surplus, G, plus premiums paid into the firm. The starting value of the liability process for each line of business is the previous year's net losses incurred. The liability growth rates are based upon the previous five year's average growth rate of the net losses and net loss adjustment expenses incurred. The volatility used to estimate the insolvency put was the previous year's volatility estimate. The methodology used to determine the implied volatility of the firm is in an appendix available from the authors.

The expected return on assets, r^i, are estimated based upon the assets classes the insurer reports on its balance sheet and in the Schedule I - Investments, both of which are available in the 10-K report. The assets classes available, the indices used to get estimates of the expected returns, and the source of the data are shown in Appendix 3. The expected returns used in each year are the annualized average return over the previous twenty four months. Premium volumes were estimated for two lines of business: short-tail lines and long-tail lines.

To estimate premium volumes, two non-linear equations, one equation for each line, need to be solved simultaneously for the two unknowns, namely short-tail and long-tail premium volume. This was done using Microsoft Excel's Solver. Starting values for the solver routine were set equal to the previous year's loss and loss adjustment expenses incurred, although the algorithm did not appear sensitive to the choice of starting values.

To evaluate the accuracy of the model of predicting premium volumes, we use the mean squared error, MSE, criterion first proposed by Theil (1966). The MSE of prediction for any year is equal to:

$$MSE = \frac{1}{J*n} \sum_{j=1}^{J} \sum_{i=1}^{n} (P_{ij} - P_{ij}^{*})^2 \tag{41}$$

where P_{ij} and P_{ij}^* are equal to the predicted and actual premium volumes, respectively, for line of business $i \in [1,n]$ in company $j \in [1,J]$. The smaller the MSE the greater the predictive power of the model.

The second measure of forecast accuracy is Theil's U statistic. Theil's U is essentially a scale free statistics. That is, it provides a relative measure of the model as opposed to the absolute error. Given the wide variation of the size of the companies in this sample, this may be important. Theil's U statistic is

$$U = \frac{\sqrt{MSE}}{\sqrt{\frac{1}{J*n}\sum_{j=1}^{J}\sum_{i=1}^{n}\left(P_{ij}^*\right)^2} + \sqrt{\frac{1}{J*n}\sum_{j=1}^{J}\sum_{i=1}^{n}\left(P_{ij}\right)^2}}. \tag{42}$$

In addition to calculating the MSE and Theil's U for premium volumes, a comparison is made between the predicted and the actual economic premium ratio for each line of business.

3.8 Accuracy Test Results

The results of the mean square error and Theil's U statistics calculations are shown in Table 3.4.[18] For a model which has perfect forecast accuracy, the value of the MSE and Theil's U would be zero. A model with no forecast accuracy will have a Theil's U of 1. The short-tail and long-tail economic premium ratio MSE statistics are .2564 and .2364 respectively. Theil's U statistic for short and long-tailed lines in .2299 and .2227 respectively. It appears as though the model estimates the economic premium ratios with just as much accuracy in both lines of business.

The MSE for premium volume in both long and short-tailed lines is very large, over 48127 for long tail lines. However, when it is scaled down using equation (42), the number become much more reasonable. Theil's U for short and long tail lines is .1244 and .0970, respectively. Given the small values for Theil's U, the model appears to be doing quite well at predicting premium volumes by line of business.

To contrast the performance of this model with other pricing models, consider the predictive power of previous financial pricing models reported in D'Arcy and Garven (1990). In their paper, they use book value data to estimate the underwriting profit

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[18] Note that the time period used here is one year shorter. This is a result of not having any net premium data by line before 1988. All observations are included except those observations which report actual negative premium volumes, those which report premium ratios greater than 5, or those for which the non-linear solver was not able to estimate the implied volatility of the firm. Of a possible 273 observations, 74 companies and 260 observations are left in the sample.

margins for the total industry from 1926 through 1985. The model which performed with the greatest accuracy over the entire sample period was a version of the total rate of return model (see Cooper (1974) and Ferrari (1968)) with a U statistic of .6840. Obviously, the methodology presented here appears to be performing quite well.

4. Conclusion

This paper proposes a financial pricing model of insurance which overcomes one of the more difficult hurdles researchers have encountered in the past: how to determine the price of insurance in each line of business that a multiple line insurance company writes. The primary reason researchers have not been able to estimate premium levels by line of business is that there did not exist an accepted way to allocate the surplus and the investment income of the insurer to the various lines of business while also considering the possibility of insolvency.

This paper uses an option pricing framework to derive a model which allows for the determination of premium levels by line of business. It is shown that the informationally competitive price of insurance for a given line of business depends on the overall risk of the firm and not solely on the risk of the individual line being priced. This rather remarkable result is due to the fact that it is not the surplus of the insurer which needs to be allocated to the various divisions of the firm, but it is the cost of insolvency which must be allocated. And because insolvencies are settled according to an equal priority rule, the individual riskiness of a given line of business is relevant to determine the premiums for that line only through its contribution to the total risk of the firm.

Empirical tests generally support the predictions of the model. In tests of price differences across insurers, it is shown that the price of insurance is inversely related to the riskiness of the firm. This inverse relationship is stronger for long- tail lines of business than for short-tail lines, suggesting that the default premium increases the longer the payout tail. A second set of tests examined price difference across lines of business within the same insurance company. After controlling for regulation, empirical support was provided for the hypothesis that the economic premium ratios across lines of business should be equal in the same insurer. A final test measured the accuracy of the model in predicting aggregate premium levels in different lines of insurance. The model was shown to be remarkably accurate at predicting premium levels by line of business.

One important avenue for future research would be to evaluate the pricing problem in a multi-period setting. The present model implicitly assumes that total incurred losses are known with certainty at the end of the policy period, but this may not be realistic for long-tail lines. Another important improvement of the model would be to incorporate catastrophic risk. Extending the model in this way may explain the relatively marginal empirical support for the second hypothesis tested in this paper. If insurers are not able to perfectly hedge catastophic risk, then these costs may be borne by policyholders in lines of business subject to this type of risks.

Bibliography

Allen, Franklin. 1993. "Estimating Divisional Cost of Capital for Insurance Companies," in J. David Cummins and Joan Lamm-Tennant, eds. *Financial Management of Life Insurance Companies.* Norwell, MA: Kluwer Academic Publishers.

Beard, R.E., T. Pentikäinen, and E. Pesonen. 1984. *Risk Theory: The Stochastic Basis of Insurance.* New York, NY: Chapman and Hall.

Berger, Larry, J. David Cummins, and Sharon Tennyson. 1992. "Reinsurance and the Liability Crisis." *Journal of Risk and Uncertainty.* 5: 253-272.

Carroll, Anne. 1993. "An Empirical Investigation of the Structure and Performance of the Private Workers' Compensation Market." *Journal of Risk and Insurance.* 60: 185-207.

Coleman, Thomas, Lawrence Fisher, and Roger Ibbotson. 1989. *U.S. Treasury Yield Curves: 1926-1988.* (New York: Moody's Investors Service).

Cooper, Robert W. 1974. "Investment Return and Property-Liability Insurance Ratemaking." *Journal of Risk and Insurance.* 45: 121-132.

Cox, John C., and Mark Rubinstein. 1985. *Options Markets.* Engelwood Cliffs, CA: Prentice Hal.

Cummins, J. David. 1988. "Risk-Based Premiums for Insurance Guaranty Funds," *Journal of Finance.* 43: 823-839.

Cummins, J. David. 1990. "Asset Pricing Models and Insurance Ratemaking," *Astin Bulletin.* 20: 125-166.

Cummins, J. David. 1992. "Financial Pricing of Property and Liability Insurance," in Georges Dionne, ed. *Contributions to Insurance Economics.* Boston, MA: Kluwer Academic Publishers.

Cummins, J. David and Patricia Danzon. 1994. "Price Shocks and Capital Flows in Liability Insurance." Working Paper, University of Pennsylvania.

D'Arcy, S. P. and James Garven. 1990. "Property-Liability Insurance Pricing Models: An Empirical Evaluation." *Journal of Risk and Insurance.* 57: 391-430.

Derrig, Richard A. 1989. "Solvency Levels and Risk Loadings Appropriate for Fully Guaranteed Property-Liability Insurance Contracts: A Financial View," in J. David Cummins, and Richard A. Derrig, eds. *Financial Models of Insurance Solvency.* Norwell, MA: Kluwer Academic Publishers.

Doherty, Neil A. and James Garven. 1986. "Price Regulation in Property-Liability Insurance." *Journal of Finance.* 41: 1031-1050.

Ferrari, J. Robert. 1968. "The Relationship of Underwriting, Investments, Leverage, and Exposure to Total Return on Owners' Equity." *Proceedings of the Casualty Actuarial Society.* 55: 295-302.

Fisher, Stanley. 1978. "Call Option Pricing when the Exercise Price is Uncertain, and the Valuation of Index Bonds." *Journal of Finance.* 33: 169-176.

Grabowski, Henry, W. Kip Viscusi, and William S. Evans. 1989. "Price and Availability Tradeoffs of Automobile Insurance Regulation." *Journal of Risk and Insurance.* 56: 275-299.

Greene, William. 1990. *Econometric Analysis.* New York: MacMillan Publishing Company.

Harrington, Scott E. 1987. "A Note on the Impact of Auto Insurance Rate Regulation." *Review of Economics and Statistics.* 69: 166-170.

Kimball, Spencer and Herbert S. Denenberg. 1969. *Insurance, Government, and Social Policy.* Homewood, IL: Richard D. Irwin, Inc.

Kneuer, Paul J. 1987. "Allocation of Surplus in a Multiline Insurer." *Discussion Paper Proceedings, Casualty Actuarial Society.* pp 191-228.

Kraus, Alan and Stephen Ross. 1982. "The Determination of Fair Profits for the Property-Liability Insurance Firm." *Journal of Finance.* 33: 1015-1028.

Lo, Andrew W. and A. Craig MacKinlay. 1990. "An Econometric Analysis of Nonsynchronous Trading." *Journal of Econometrics.* 45: 181-211.

Litterman, Robert and Thomas Iben. 1991. "Corporate Bond Valuation and the Term Structure of Credit Spreads." *Journal of Portfolio Management.* v17n3: 52-64.

Merton, Robert. 1974. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance.* 29: 449-470.

Modigliani, F. and M.H. Miller. 1958. "The Cost of Capital, Corporation Finance, and the Theory of Investment." *American Economic Review.* 48: 261-297.

Myers, Stewart and Richard Cohn. 1987. "Insurance Rate Regulation and the Capital Asset Pricing Model," in J. David Cummins and Scott Harrington, eds. *Fair Rate of Return in Property-Liability Insurance.* Norwell, MA: Kluwer Academic Publishers.

Pitts, Mark and Frank J. Fabozzi. 1991. "Fixed Income Options and Futures Markets," in Frank J. Fabozzi, ed. *Handbook of Fixed Income Securities.* Homewood, IL: Richard D. Irwin, Inc.

Theil, Henri. 1966. *Applied Economic Forecasting.* Amsterdam: North Holland.

TABLE 3.1: Summary Statistics For Price Differences Tests

Variable	Symbol	Number of Obs.	Mean	Standard Deviation	Min	Max
Long-tailed PR	LPR	348	1.376	0.391	0.509	3.579
Long-tailed PR controlling for expenses	LPRE	348	0.940	0.231	0.355	2.377
Long-tailed EPR	LEPR	348	1.650	0.476	0.615	4.310
Long-tailed EPR controlling for expenses	LEPRE	348	1.128	0.288	0.412	2.862
Short-tailed PR	SPR	348	1.814	0.616	0.429	4.587
Short-tailed PR controlling for expenses	SPRE	348	1.177	0.443	0.162	4.779
Short-tailed EPR	SEPR	348	1.918	0.661	0.461	4.905
Short-tailed EPR controlling for expenses	SEPRE	348	1.244	0.467	0.174	4.997
Standard deviation of equity	SIGMAE	348	0.332	0.226	0.070	2.165
Equity (000's)	EQUITY	348	1,301,804.22	2,833,940.27	747.00	24,541,308.00
Market Value of Assets (000's)	MVASSETS	337	7,509,577.20	16,380,501.25	12,507.56	90,252,428.64
Total Liabilities (000's)		348	6,257,859.24	14,840,875.50	7,333.80	84,603,100.00

TABLE 3.2: Price Equation
Controlling for Year and Company Random Effects

Dependent Variable	Intercept	SIGMAE	NOTRADES	LMVA	Variance Component for Cross Sections	Variance Component for Time Series	Variance Component for Error	No. of Firms
LPR	2.1234 ***	-0.1799 **	0.0003	-0.0504 ***	0.0699	0.0029	0.0372	58
	0.2925	0.0923	0.0006	0.0194				
LPRE	1.4705 ***	-0.1609 ***	-0.0003	-0.0338 ***	0.0175	0.0014	0.0148	58
	0.1583	0.0571	0.0003	0.0104				
LEPR	2.5755 ***	-0.2455 **	0.0001	-0.0615 *	0.1016	0.1034	0.0572	58
	0.3605	0.1153	0.0007	0.0237				
LEPRE	1.7310 ***	-0.2027 ***	-0.0005	-0.0379 ***	0.0284	0.0049	0.0227	58
	0.2021	0.0714	0.0004	0.0132				
SPR	2.6666 ***	-0.2251	-0.0006	-0.0555 *	0.1245	0.0111	0.1542	58
	0.4449	0.1787	0.0009	0.0291				
SPRE	1.6700 ***	-0.1333	-0.0004	-0.0322	0.0465	0.0053	0.1043	58
	0.3032	0.1411	0.0007	0.0197				
SEPR	2.8442 ***	-0.2415	-0.0007	-0.0603 *	0.1492	0.0155	0.1729	58
	0.4823	0.1901	0.0010	0.0315				
SEPRE	1.7842 ***	-0.1474	-0.0005	-0.0352 *	0.0516	0.0071	0.1152	58
	0.3197	0.1486	0.0007	0.0207				

notes: *** - significant at 1 % level
** - significant at 5 % level
* - significant at 10 % level

TABLE 3.3
Price Differences within the Same Insurer

T-Test

	Mean	Standard Deviation	Number of Observations	T-statistic H0: x = 1	Prob |T| > 0
All Lines	1.1425	0.4417	339	5.940	0.00%

Fixed Effects Regression

	R^2	F-Statistic	Intercept (α)	Standard Error	T-statistic H0: α=1	Prob |T| > 0
All Lines	0.3782	2.5680	1.1648	0.0733	2.249	2.51%

Random Effects Regression

	Variance Component for Cross Sections	Variance Component for Time Series	Variance Component for Error	Intercept (α)	Standard Error	T-statistic H0: α=1	Prob |T| > 0
All Lines	0.0462	0.0048	0.0995	1.1351	0.0459	2.942	0.35%

Price Differences within the Same Insurer
Eliminating Highly Rate Regulated Lines of Business

T-Test

	Mean	Standard Deviation	Number of Observations	T-statistic H0: x = 1	Prob |T| > 0
Unregulated Lines	1.1446	0.6346	330	4.140	0.00%

Fixed Effects Regression

	R^2	F-Statistic	Intercept (α)	Standard Error	T-statistic H0: α=1	Prob |T| > 0
Unregulated Lines	0.5717	5.0300	1.0990	0.0904	1.095	27.43%

Random Effects Regression

	Variance Component for Cross Sections	Variance Component for Time Series	Variance Component for Error	Intercept (α)	Standard Error	T-statistic H0: α=1	Prob |T| > 0
Unregulated Lines	0.0736	0.0072	0.1418	1.0927	0.0566	1.637	10.28%

TABLE 3.4: Mean Squared Error and Theil's U Statistics

Variable	Number of Obs.	Mean Squared Error	Standard Deviation of Squared Error	Minimum Squared Error	Maximum Squared Error	Theil's U
Short-tail line premiums (in $100 b's)	260	3,091.34	8,510.83	0.00	81,425.00	0.12
Long-tail lines premiums (in $100 b's)	260	48,127.04	172,219.61	0.00	1,430,739.20	0.10
Short-tail lines economic premium ratio	260	0.26	0.57	0.00	6.46	0.23
Long-tail lines economic premium ratio	260	0.24	0.99	0.00	10.15	0.22

Technical Appendix

This appendix will develop a financial pricing model which can be used to value the options of a two-line insurance company. The analysis to n lines of business is straight forward.

Assume there are two time periods, time 0 and time 1. The insurance company consists of two lines of business and equityholders. At time 0 premiums of P_i are collected from policyholders for line i, where i=1,2. The equityholders of the firm contribute surplus of G. Let $A(0)=P_1+P_2+G$ be the market value of the assets of the company at time 0. Because of imperfect contracts, the premiums and the surplus are all paid at time 0 to avoid the possibility of nonpayment after the losses have been realized at time 1. In return for premiums, the insurance company agrees to underwrite the expected liability payments for each line of business, L_i.

The premiums for each line of business and the surplus will be invested. Assume the market value of the premiums, surplus and liabilities evolve according to the following stochastic processes:

$$dP_i = \alpha_{P_i} P_i dt + \sigma_{P_i} P_i dz_{P_i} \tag{A.1}$$

$$dG = \alpha_G G dt + \sigma_G G dz_G \tag{A.2}$$

$$dL_i = \mu_i L_i dt + \sigma_{L_i} L_i dz_{L_i} \tag{A.3}$$

where P_i, G, L_i = invested premiums, invested surplus, and liabilities for line i, respectively,

α_i, α_G, μ_i = instantaneous drift on invested premiums and liabilities for line i

σ_{P_i}, σ_G, σ_{L_i} = instantaneous standard deviation of invested premiums, invested surplus, and liabilities for line i, and

dz_{P_i}, dz_G, and dz_{L_i} = standard diffusion process (Wiener process).

The instantaneous correlation coefficients between the diffusion processes are as follows: $dz_{P_i} z_G = \rho_{GP_i}$ dt for i=1,2, $dz_{P_i} dz_{P_j} = \rho_{P_iP_j}$ dt for i≠j, $dz_{L_i} dz_{P_j} = \rho_{L_iP_j}$ dt for i=1,2 and j=1,2, $dz_{L_i} dz_G = \rho_{GL_i}$ dt for i=1,2, $dz_{L_i} dz_{L_j} = \rho_{L_iL_j}$ dt for i≠j.

Both assets and liabilities are assumed to be priced according to an intertemporal asset pricing model, such as the intertemporal capital asset pricing model (ICAPM). The ICAPM implies the following return relationships:

$\alpha_j = r_f + \pi_j$, for j= P_1, P_2, G

$\mu_{L_i} = r_{L_i} + \pi_{L_i}$, for liability classes i=1,2.

where r_{L_i} = inflation rate in liability class i, and

π_i = the market risk premium for asset i = P_1, P_2, G, L_1, L_2.

Since we have assumed that the invested assets and liabilities are priced according to the ICAPM, the risk premium, π_i, would be

$\pi_i = \rho_{iM}(\sigma_i/\sigma_M)[\mu_M - r_f]$

where μ_M, σ_M are the drift and diffusion parameters of the market portfolio and ρ_{iM} is the correlation coefficient between the Brownian motion process for asset or liability i and the market portfolio.

The value of any divisional option, either the divisional call option or the divisional put option, can be written as $H_i(P_i,L_i,\tau)$ where τ is the time to expiration of the option. Differentiating H_i using Ito's lemma and invoking the ICAPM pricing relationships for the premiums and liabilities yields

$$Hr_f = r_f H_{P_i} P_i + r_{L_i} H_{L_i} L_i - H_\tau + \frac{1}{2}\sigma_{P_i}{}^2 H_{P_i P_i} + H_{P_i L_i} P_i L_i \rho_{P_i L_i} \sigma_{P_i} \sigma_{L_i} + \frac{1}{2}\sigma_{L_i}{}^2 H_{L_i L_i} \qquad \text{(A.4)}$$

The risk , i.e. any term multiplied by a dz_j term, and the market risk parameters, i.e., the π_j terms, have been eliminated by using the ICAPM and taking expectations. It is also possible to do this by using a hedging argument, such as the one used by Fischer (1978). However, this assumes that the appropriate hedging securities are available, which may not be the case with non-traded liability contracts.

The next step is to use the homogeneity property of the options model to change variables so that the model is expressed in terms of the premium-to-liability ratio, $x_i=P_i/L_i$, and the option value-to-liability ratio $h_i=H_i/L_i$. The result is the following differential equation:

$$(r_f - r_{L_i})h^i = (r_f - r_{L_i})x_i h^i_{x_i x_i} + \tfrac{1}{2}(\sigma^2_{P_i} + \sigma^2_{L_i} - 2\sigma_{P_i}\sigma_{L_i}\rho_{P_i L_i})h^i_{x_i x_i} x_i^2 - h^i_\tau \qquad \text{(A.5)}$$

where r_f = risk-free rate of interest

r_{L_i} = inflation component of the instantaneous liability drift term,

$h^i = H_i(P_i,L_i,\tau)/L_i$, and

$x_i = P_i/L_i$.

Equation (A.5) is the standard Black-Scholes differential equation, where the optioned asset is the premium-to-liability ratio for line i. To obtain the value of any specific claim on a division of the firm one would solve (A.5) subject to the appropriate boundary conditions. For example, the value of the option held by the equityholders of the firm which entitles them to the residual value of the division after all claims have been paid can be modeled as a call option. The boundary conditions for this option are $C_i(0,L_i,\tau) = 0$, and $C_i(P,L_i,\tau)=MAX(P_i-L_i,0)$.

The process to find the value of any contingent claim on the entire firm is very similar to the methodology used to determine the value of the divisional options. The value of an option on the entire two-line insurance company can be written as $H(P_1,P_2,G,L_1,L_2,\tau)$. Differentiating H using Ito's lemma and invoking the ICAPM pricing relationships yields

$$
\begin{aligned}
Hr_f &= r_f H_{P_1} P_1 + r_f H_{P_2} P_2 + r_f H_G G + r_{L_1} H_{L_1} L_1 + r_{L_1} H_{L_1} L_1 - H_\tau \\
&+ \frac{1}{2}\sigma_{P_1}{}^2 H_{P_1P_1} + \frac{1}{2}\sigma_{P_2}{}^2 H_{P_2P_2} + \frac{1}{2}\sigma_G{}^2 H_{GG} + \frac{1}{2}\sigma_{L_1}{}^2 H_{L_1L_1} + \frac{1}{2}\sigma_{L_2}{}^2 H_{L_2L_2} + H_{P_1L_1} P_1 L_1 \rho_{P_1L_1} \sigma_{P_1} \sigma_{l} \\
&+ H_{P_1L_2} P_1 L_2 \rho_{P_1L_2} \sigma_{P_1} \sigma_{L_2} + H_{P_2L_2} P_2 L_2 \rho_{P_2L_2} \sigma_{P_2} \sigma_{L_2} + H_{P_2L_1} P_2 L_1 \rho_{P_2L_1} \sigma_{P_2} \sigma_{L_1} + H_{P_1P_2} P_1 P_2 \rho_{P_1P_2} \\
&+ H_{GP_1} G P_1 \rho_{GP_1} \sigma_G \sigma_{P_1} + H_{GP_2} G P_2 \rho_{GP_2} \sigma_G \sigma_{P_2} + H_{GL_1} G L_1 \rho_{GL_1} \sigma_G \sigma_{L_1} + H_{GL_2} G L_2 \rho_{GL_2} \sigma_G \sigma_{L_2}
\end{aligned}
\tag{A.6}
$$

Note, the risk and drift parameters have again been eliminated by using the ICAPM pricing relationships and taking expectations. The next step is to use the homogeneity property of the options model again. This time we want to express the model in terms of the asset-to-liability ratio x=A/L where $A=P_1+P_2+G$ and $L = L_1+L_2$, and in terms of the option-to-liability ratio h=H/L and the liability and asset proportions, i.e. $w_{L_i} = L_i/L$ and $w_{P_i} = P_i/A$ and $w_G = G/A$. Note, this requires us to make the assumption that the sum of lognormally distributed random variables are lognormally distributed, e.g., that L_1+L_2 can be approximated by a lognormal diffusion process. The assumption about the additivity of lognormals is routinely used in the discrete time option pricing literature (e.g., Stapleton and Subramanyam (1984)). The result is the following differential equation

$$
r_x h = r_x x h_x + \frac{1}{2} x^2 h_{xx} \sigma_x - h_\tau \tag{A.7}
$$

where $r_x = r_f - w_{L_1} r_{L_1} - w_{L_2} r_{L_2}$

$$
\begin{aligned}
\sigma_x{}^2 = {} & w_G{}^2\sigma_G{}^2 + w_{P_1}^2 \sigma_{P_1}^2 + w_{P_2}^2 \sigma_{P_2}^2 + w_{L_1}^2 \sigma_{L_1}^2 + w_{L_2}^2 \sigma_{L_2}^2 - 2 w_{P_1} w_{L_1} \sigma_{P_1} \sigma_{L_1} \rho_{P_1L_1} - \\
& 2 w_{P_1} w_{L_2} \sigma_{P_1} \sigma_{L_2} \rho_{P_1L_2} - 2 w_{P_2} w_{L_1} \sigma_{P_2} \sigma_{L_1} \rho_{P_2L_1} - 2 w_{P_2} w_{L_2} \sigma_{P_2} \sigma_{L_2} \rho_{P_2L_2} - \\
& 2 w_G w_{L_1} \sigma_G \sigma_{L_1} \rho_{GL_1} - 2 w_G w_{L_2} \sigma_G \sigma_{L_2} \rho_{GL_2} + 2 w_G w_{P_1} \sigma_G \sigma_{P_1} \rho_{GP_1} + \\
& 2 w_G w_{P_2} \sigma_G \sigma_{P_2} \rho_{GP_2} + 2 w_{P_1} w_{P_2} \sigma_{P_1} \sigma_{P_2} \rho_{P_1P_2} + 2 w_{L_1} w_{L_2} \sigma_{L_1} \sigma_{L_2} \rho_{L_1L_2}
\end{aligned}
$$

$A = P_1 + P_2 + G$
$L = L_1 + L_2$
$h = H(A,L,\tau)/L$
$x = A/L$
$w_{P_i} = P_i/A$, for i = 1,2
$w_G = G/A$
$w_{L_i} = L_i/L$, for i = 1,2

Equation (A.7) is the standard Black-Scholes differential equation, where the optioned asset is the asset-to-liability ratio of the entire firm, x.

Appendix 1
Publicly Traded Property-Liability
Companies Included in the Empirical Tests

Aetna
ALFA Corp
Allied Group Inc
Allmerica Property and Casualty
American Bankers Ins. Group
American Indemnity Financial
American International Group
Argonaut Group Inc
AVEMCO Corp
Baldwin & Lyons
Bancinsurance
Berkley (WR) Corp
Berkshire Hathaway
Cigna
CNA
Capitol Transamerica
Chubb
Cinncinnati Financial
Citation Insurance Group
Citizens Security Group
Condor Services Inc
Continental Corp
Danielson Holding Corp
Donegal Insurance Group
EMC Insurance Group
First Central Financial
Foremost Corp of America
Fremont General
Frontier Insurance
GEICO
GAINSCO
General RE Corp
Hartford Steam and Boiler
Independent Ins. Group
Intercargo Corp
Kemper Corp
Lawrence Insurance Group
Lincoln National
Merchants Group
Mercury General Corp
Meridian Ins Group
Milwaukee Ins Group
Mobile America Corp
Nac RE Corp
Nymagic Corp
National RE
National Security Corp
Navigators Group
North East Ins Co
Ohio Casualty
Old Republic International
Orion Capital Corp.
Phoenix RE Corp
Piedmont Management
Progressive Corp
RLI Corp
RE Capital Corp
Reliance Corp
Riverside Group
Scor US Reinsurance
SAFECO
St. Paul Companies
Seibels Bruce Group
Selective Insurance Group
State Auto Financial Corp
Transamerica
Transatlantic Holding
Travelers Corp
Trenwick Group
20th Century Industries
USFG
United Fire and Casualty
United State Facilities Co
Unitrin Inc
Victoria Financial
Walshire Assurance
Zenith National Insurance

Appendix 2
Short and Long Tailed Lines of Business Used in this Study

Short-Tail Lines	Long-Tail Lines
Fire	Farmowners Multiple Peril
Allied Lines	Homeowners Multiple Peril
Mortgage Guaranty	Commercial Multiple Peril
Inland Marine	Ocean Marine
Financial Guaranty	Medical Malpractice
Earthquake	International
Fidelity	Reinsurance
Surety	Workers Compensation
Glass	Other Liability
Burglary and Theft	Products Liability
Credit	Aircraft
Automobile Physical Damage	Boiler and Machinery
	Automobile Liability

Appendix 3
Asset Classes and the Indicies
Used to Determine the Expected Return on Assets

Asset Class	Index
U.S. Gov't Bonds	Long Term Gov't Total Return Bond Index - Ibottson
State Municipal Bonds	Long Term Corporate Total Return Bond Index - Ibottson
Foreign Gov't Bonds	Non-U.S. Gov't Total Return Bond Index - Salomon Bros.
Public Utility Bonds	Long Term Corporate Total Return Bond Index - Ibottson
Convertible Warrants	S&P 500 - Ibottson
Mortgage Back Securities	30 Yr. GNMA Total Return Index - Salomon Bros.
All other Corporate Bonds	Long Term Corporate Total Return Bond Index - Ibottson
Certificates of Deposit	30 Day T-Bill Total Return Index - Ibottson
Non-Redeemable Preferred Stocks	S&P Preferred Stock Total Return Index - S&P
Commons Stocks	S&P 500 - Ibottson
Short-term Investments	1 Yr. T-Bill Total Return Index - Ibottson
Mortgages	Total Return Mortgages Index - Salomon Bros.
Real Estate	S&P REIT Total Return Index - S&P
Policy Loans	Intermediate U.S. Gov't Total Return Bond Index - Ibottson
Cash	none
Accrued Investment Income	1 Yr. T-Bill Total Return Index - Ibottson
Deferred Acquisition Costs	1 Yr. T-Bill Total Return Index - Ibottson
Other Assets	1 Yr. T-Bill Total Return Index - Ibottson