A SHORTFALL APPROACH TO THE EVALUATION OF RISK AND RETURN OF POSITIONS WITH OPTIONS

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Abstract

The authors propose a general shortfall approach to evaluating risk resp. risk and return of stock positions with options to take proper account of the typical asymmetry of the risk-return-profile of such a position. Two shortfall-performance measures are proposed as well. In addition an explicit analytical derivation of expressions for shortfall-probability, shortfall-expectation and shortfall-semivariance of the collar position (which in turn contains the pure stock position, the put hedge position and the covered-short-call-position as special cases) is performed in case of a normal resp. lognormal distribution of the underlying stock position.

Keywords

shortfall risk; shortfall performance; equity positions with options; collar

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Résumé

Les auteurs proposent une approche analytique de shortfall en vue d'evaluer le risque et le rendement d'une investissement en actions en combinaison avec une option "long-put" et une option "short-call" (collar). Les mesures du risque utilisées dans l'analyse tiennent particulièrement compte de l'asymétrie de cet investissement avec options. Dans le cas où l'investissement en actions est distribué normale ou lognormale les auteurs calculent les mesures shortfall-probabilité, shortfall-moyenne et shortfall-semivariance pour la stratégie collar. Cette stratégie inclut l'investissement uniquement en actions, le put-hedge et le covered-short-call.

1. Introduction

The evaluation of the risk resp. the risk-return-profile of stock positions containing options has been - beginning with a series of papers by BOOKSTABER/CLARKE (1983a, b; 1984, 1985) - the topic of a number of contributions, cf. EVNINE/RUDD (1984), LEWIS (1990), FERGUSON (1993), MARMER/NG (1993) and LEE (1994) .

The typical asymmetrical profile of positions with options turns out to be the central problem and an application of traditional risk measures, like variance or standard deviation, which are symmetrical of nature, is very questionable. For example in case of a 1:1 put hedge the "downside volatility" is absolutely limited. On the other hand the investor participates from increasing prices of the underlying object to an unlimited extent, the participation only being reduced by the option premium. Variance resp. standard deviation in this case would be measures for investor's chances rather than measures of risk. Regarding this asymmetrical nature the obvious thing to do would be to measure the risk of positions with options by a measure of an asymmetrical nature, too. A general class of such measures has been developed by ALBRECHT (1994) containing the risk measures shortfall-probability, shortfall-expectation as well as shortfall-semivariance as special cases. The use of shortfall-semivariance resp. its square-root, the shortfall semi-standard deviation, is being proposed by LEWIS (1990) and MARMER/NG (1993) as an adequate measure of risk in case of positions with options, too. However, these authors do not carry out a complete analytical investigation of their approach. In the present paper in contrast to that a general analytical approach based on the general shortfall conception of ALBRECHT (1994) is developed. Subsequently the three mentioned specific risk measures are calculated for the collar position, which in turn contains the pure stock position, the put-hedge and the covered-short-call as special cases. The calculation is based on the assumption of a normally or lognormally distributed price process of the underlying stock position.

2. A general shortfall-approach for the evaluation of positions with options

We fix a time interval $[0, T]$ and let denote $\{V_t\ ;\ 0 \leq t \leq T\}$ the development of the value of the analysed position, in the present case a combined position in stocks and options. The objective of the present paper now is the evaluation of the positon V_T on the basis of the general shortfall approach as developed in ALBRECHT (1994). However, in contrast to ALBRECHT (1994) we do not analyse shortfall returns but shortfall positions on an absolute monetary level. Specifying a minimum final wealth positon $m = m(T)$, we postulate the condition

$$V_T \geq m\ . \tag{2.1}$$

We now define the *shortfall-position of final wealth* by

$$V_T^-(m) = \max(m - V_T, 0) \ . \tag{2.2}$$

This definition can obviously be used for the analysis of an *arbitrary* wealth positon but in the following we only look at special combined positions of stocks and options.

Let denote $\{S_t;\ 0 \le t \le T\}$ the price process of a single stock or a portfolio of stocks, defining the underlying object for the options considered. Let Y denote the exercise price of a call option with maturity date T on one unit of the underlying object and let $\{C_t = C_t(Y)\ ;\ 0 \le t \le T\}$ denote the price process for that option. Let then X denote the exercise price of a put option with maturity date T on one unit of the underlying object and let denote $\{P_t = P_t(X);\ 0 \le t \le T\}$ the corresponding price process.

Now a *1:1 collar position* is defined by simultaneously holding the underlying object, buying a put with exercise price X and selling a call with exercise price $Y > X$. Defining $\Delta(X, Y) := P_0(X) - C_0(Y)$ we therefore have for the final wealth position of the 1:1 collar:

$$\begin{aligned} V_T &= S_T + \max(X - S_T, 0) - \max(S_T - Y, 0) - \Delta(X, Y) \\ &= \min[\max(S_T, X), Y] - \Delta(X, Y) \ . \end{aligned} \tag{2.3}$$

The corresponding shortfall-position of final wealth is given by

$$V_T^-(m) = \max\{m - \min[\max(S_T, X), Y] + \Delta(X, Y), 0\} . \quad (2.4)$$

The evaluation of this position is the main objective of the paper. It has to be noted, however, that the collar position contains as special cases the put hedge position for $Y \to \infty$, the covered short call position for $X \to -\infty$ as well as the pure stock position for $X \to -\infty$ and $Y \to \infty$. In addition pure option positions, combined option positions (straddle, strangle, etc.) as well as other combined positions in stocks and options can be analysed on the basis of (2.2) as well.

A general approach, cf. ALBRECHT (1994), for the evaluation of the position $V_T^-(m)$ requires the introduction of a loss functional L and the determination of the expected loss of the final shortfall-position:

$$SR_m(V_T) := \mathrm{E}[L(V_T^-(m))] . \quad (2.5)$$

On the basis of $L(x) = x^n$, $n \in \mathbb{N}_0$ we obtain a number of important measures for shortfall-risk. We obtain (in the case n = 0 we define $\max(m - V_T, 0)^0 := I_{(-\infty, m)}(V_T)$)

$$\begin{aligned} SR_m(V_T) &:= \mathrm{E}[V_T^-(m)^n] \\ &= \mathrm{E}[\max(m - V_T, 0)^n] = LPM_m^n(V_T) . \end{aligned} \quad (2.6)$$

The resulting measures of shortfall risk are identical to the lower partial moments of the random variable V_T. This implies that results on the calculation of partial moments, cf. e.g. WINKLER et al. (1972) can be used for the specific calculation of these measures. As special cases for $n = 0, 1, 2$ we obtain the well known risk measures shortfall-probability, shortfall-expectation as well as shortfall-semivariance:

$$SP_m(V_T) = \mathrm{E}[V_T^-(m)^0] = \mathrm{P}(V_T \leq m) \ , \tag{2.7a}$$

$$SE_m(V_T) = \mathrm{E}[\max(m - V_T, 0)] \ , \tag{2.7b}$$

$$SSV_m(V_T) = \mathrm{E}[\max(m - V_T, 0)^2] \ . \tag{2.7c}$$

In the following we focus on the analytical calculation of (2.7a - c) for the collar position (2.4) (containing the put hedge, the covered short call and the pure stock position as special cases), i.e. we have to calculate the following figures:

$$\mathrm{P}[\min\{\max(S_T, X), Y\} - \Delta(X, Y) \leq m] \ , \tag{2.8a}$$

$$\mathrm{E}[\max\{m - \min[\max(S_T, X), Y] + \Delta(X,Y), 0\}] \ , \tag{2.8b}$$

$$\mathrm{E}[\max\{m - \min[\max(S_T, X), Y] + \Delta(X,Y), 0\}^2] \ . \tag{2.8c}$$

To do this we have to make an assumption with respect to the distribution of S_T.. In case of the standard assumption, that $\{S_t; 0 \leq t \leq T\}$ follows a geometrical Brownian motion process with constant drift u and constant volatility s we have, cf. HULL (1993, p. 210):

$$\ln S_T \sim N(\mu, \sigma^2) \, , \tag{2.9a}$$

where

$$\mu = \ln s_0 + (u - \frac{s^2}{2})T \, , \, \sigma^2 = s^2 T \, . \tag{2.9b}$$

As a consequence the first special case for the distribution of S_T we consider is the case (2.9) of a lognormal distribution with parameters μ and σ^2. As for short time intervals T the logarithmic normal distribution can be reasonably approximated by a normal distribution, we consider a normally distributed S_T as an alternative special case.

The present paper focusses on the evaluation of shortfall risk of positions with options. However, when doing a portfolio selection, we have to calculate in addition the expected value of the position and to carry through a trade off between shortfall risk (typically shortfall-expectation or shortfall-semivariance) and expected value. For these purposes we calculate in addition the expected value of the collar position and - for the sake of completeness- the variance of the collar position, too.

To be able to compare the perfomance of alternative positions with options an obvious thing to do would be to modify the SHARPE ratio $T(R) = [E(R) - r_0]/\sigma(R)$ by replacing the risk measure $\sigma(R)$ by a corresponding measure $SR_m(R)$ of shortfall risk. In the present case, where we do the analysis on an absolute monetary level, simple *shortfall-performance measures* would be:

$$T_1^m(V_T) := \frac{\mathrm{E}(V_T)}{SE_m(V_T)} \quad \text{resp.} \quad T_2^m(V_T) := \frac{\mathrm{E}(V_T)}{\sqrt{SSV_m(V_T)}} \,. \tag{2.10}$$

In case of $SE_m(V_T) = 0$ resp. $SSV_m(V_T) = 0$ performance could be e.g. compared on the basis of $\mathrm{E}(V_T)$ alone.

3. General results

The detailed position of the collar is given according to (2.3) by ($V = V_T$, $S = S_T$, $\Delta = \Delta(X, Y)$:

$$V = \begin{cases} X - \Delta & S \leq X \\ S - \Delta & X < S < Y \\ Y - \Delta & S \geq Y . \end{cases}$$

Defining F to be the distribution function of S and assuming that S possesses a density function f, then V possesses the following distribution:

$$P(V < X - \Delta) = 0$$

$$P(V = X - \Delta) = F(X)$$

$$P(a \leq V \leq b) = \int_{a-\Delta}^{b-\Delta} f(x)\, dx \; ; X - \Delta < a < b < Y - \Delta$$

$$P(V = Y - \Delta) = 1 - F(Y)$$

$$P(V > Y - \Delta) = 0 .$$

For the expected value consequently we obtain

$$\mathrm{E}(V) = X\,\mathrm{F}(X) + \int_X^Y x f(x)\,dx + Y(1 - \mathrm{F}(Y)) - \Delta\ . \tag{3.1}$$

For the variance we similarly obtain:

$$\begin{aligned}\mathrm{Var}(V) &= \mathrm{Var}(V + \Delta) = \mathrm{E}(V + \Delta)^2 - \left[\mathrm{E}(V + \Delta)\right]^2 \\ &= X^2\,\mathrm{F}(X) + \int_X^Y x^2 f(x)\,dx + Y^2(1 - \mathrm{F}(Y)) \\ &\quad - \left[X\,\mathrm{F}(X) + \int_X^Y x f(x)\,dx + Y(1 - \mathrm{F}(Y))\right]^2 .\end{aligned} \tag{3.2}$$

It is obvious that the measures of shortfall risk (2.8a - c) are depending on the magnitude of the target value m compared to X - Δ resp. Y - Δ. Thus we have three cases:

Case 1 : $m \leq X - \Delta$

In this case we have $\mathrm{P}(V \geq m) = 1$ and therefore $V^-(m) \equiv 0$.
The distribution of $V^-(m)$ is thus given by:
$\mathrm{P}(V^-(m) = 0) = 1$
$\mathrm{P}(V^-(m) > 0) = 0$.
Consequently we have $SP_m = SE_m = SSV_m = 0$ in this case.

Case 2: $X - \Delta < m \leq Y - \Delta$

In this case we have

$$V^{-}(m) = \begin{cases} m - X + \Delta & S \leq X \\ m - S + \Delta & X < S < m + \Delta \\ 0 & S \geq m + \Delta\,. \end{cases}$$

For the distribution of $V^{-}(m)$ we consequently obtain:

$$\mathrm{P}(V^{-}(m) = 0) = \mathrm{P}(S \geq m + \Delta) = 1 - \mathrm{F}(m + \Delta)$$

$$\mathrm{P}(a \leq V^{-}(m) \leq b) = \int_{m-b+\Delta}^{m-a+\Delta} f(x)\,dx; \quad 0 < a < b < m - X + \Delta$$

$$\mathrm{P}(V^{-}(m) = m - X + \Delta) = \mathrm{F}(X)$$

$$\mathrm{P}(V^{-}(m) > m - X + \Delta) = 0\,.$$

Therefore we obtain for the shortfall probability:

$$\mathrm{P}(V^{-}(m) > 0) = \mathrm{P}(S < m + \Delta) = \mathrm{F}(m + \Delta)\,, \tag{3.3}$$

and for the shortfall-expectation

$$
\begin{aligned}
\mathrm{E}[V^-(m)] &= (m - X + \Delta)\mathrm{F}(X) + \int_X^{m+\Delta} (m - x + \Delta) f(x)\, dx \\
&= (m + \Delta)\mathrm{F}(m + \Delta) - X\mathrm{F}(X) - \int_X^{m+\Delta} x f(x)\, dx
\end{aligned}
\tag{3.4}
$$

and finally for the shortfall-semivariance:

$$
\begin{aligned}
\mathrm{E}[V^-(m)^2] &= (m - X + \Delta)^2 \mathrm{F}(X) \\
&\quad + \int_X^{m+\Delta} (m - x + \Delta)^2 f(x)\, dx \; .
\end{aligned}
\tag{3.5}
$$

Case 3: $m > Y - \Delta$

In this case we have $\mathrm{P}(V < m) = 1$ and therefore $V^-(m)$ is identical to $m - V$. For the distribution of $V^-(m)$ we consequently obtain:

$\mathrm{P}(V^-(m) < m - Y + \Delta) = 0$

$\mathrm{P}(V^-(m) = m - Y + \Delta) = 1 - \mathrm{F}(Y)$

$$P(a \leq V^-(m) \leq b) = \int_{m-a+\Delta}^{m-b+\Delta} f(x)\,dx; \; m - Y + \Delta < a < b < m - X + \Delta$$

$$P(V^-(m) = m - X + \Delta) = F(X)$$

$$P(V^-(m) > m - X + \Delta) = 0\ .$$

Obviously we have for the shortfall-probability

$$P(V^-(m) > 0) = 1\ .$$

For the shortfall-expectation $E(V^-(m))$ we obtain:

$$\begin{aligned} E[V^-(m)] &= m - E(V) \\ &= m - X\,F(X) - \int_X^Y x f(x)\,dx - Y(1 - F(Y)) + \Delta \end{aligned} \tag{3.6}$$

and finally we have for the shortfall-semivariance:

$$\begin{aligned} E[V^-(m)^2] &= (m - Y + \Delta)^2(1 - F(Y)) \\ &\quad + \int_X^Y (m - x + \Delta)^2 f(x)\,dx + (m - X + \Delta)^2\,F(X)\ . \end{aligned} \tag{3.7}$$

4. Results in case of special distributions of the stock price

4.1 Normal distribution

Let S follow a normal distribution with parameters μ and σ^2 and let denote Φ the distribution function and φ the density function of a standard normal distribution. In addition we define $x_N := (x - \mu)/\sigma$ for an arbitrary $x \in \mathrm{IR}$. As some of the calculations are quite tedious, we will in the following only present the results (a more detailed version is ALBRECHT et al. (1994), which can be obtained upon request by the first author).

For the expected value we obtain:

$$\begin{aligned} E(V) &= X\Phi(X_N) + \sigma[\varphi(X_N) - \varphi(Y_N)] \\ &\quad + \mu[\Phi(Y_N) - \Phi(X_N)] + Y[1 - \Phi(Y_N)] - \Delta \end{aligned} \tag{4.1}$$

For the variance we obtain:

$$\begin{aligned} \mathrm{Var}(V) &= X^2\Phi(X_N) + (\mu^2 + \sigma^2)[\Phi(Y_N) - \Phi(X_N)] \\ &\quad + \sigma(\mu + X)\varphi(X_N) - \sigma(\mu + Y)\varphi(Y_N) + Y^2[1 - \Phi(Y_N)] \\ &\quad - \{X\Phi(Y_N) + \sigma[\varphi(X_N) - \varphi(Y_N)] \\ &\quad + \mu[\Phi(Y_N) - \Phi(X_N)] + Y[1 - \Phi(Y_N)]\}^2 . \end{aligned} \tag{4.2}$$

Using the notation $M_N = (m + \Delta)_N = (m + \Delta - \mu) / \sigma$ we obtain for the shortfall probability in case of $X - \Delta < m \leq Y - \Delta$:

$$P(V^-(m) > 0) = \Phi(M_N) \ . \tag{4.3}$$

For the shortfall-expectation we obtain:

$$\begin{aligned} E[V^-(m)] &= (m + \Delta)\Phi(M_N) - X\Phi(X_N) \\ &\quad - \mu[\Phi(M_N) - \Phi(X_N)] + \sigma[\varphi(M_N) - \varphi(X_N)] \ . \end{aligned} \tag{4.4}$$

Finally we obtain for the shortfall-semivariance in this case:

$$\begin{aligned} E[V^-(m)^2] &= (m - X + \Delta)^2\Phi(X_N) \\ &\quad + [(\mu - m - \Delta)^2 + \sigma^2][\Phi(M_N) - \Phi(X_N)] \\ &\quad + \sigma[m + \Delta - \mu]\ \varphi(M_N) \\ &\quad - \sigma[2(m + \Delta) - \mu - X]\ \varphi(X_N) \ . \end{aligned} \tag{4.5}$$

In case $m > Y - \Delta$ we obtain for the shortfall-expectation:

$$\begin{aligned} E[V^-(m)] &= m - X\Phi(X_N) - \sigma[\varphi(X_N) - \varphi(Y_N)] \\ &\quad - \mu[\Phi(Y_N) - \Phi(X_N)] - Y[1 - \Phi(Y_N)] + \Delta \ . \end{aligned} \tag{4.6}$$

For the shortfall-semivariance we finally obtain in this case:

4.2 Logarithmic normal distribution

Now let S_T follow the lognormal distribution (2.9). We define $x_{LN} := (\ln x - \mu)/\sigma$ for an arbitrary $x \in$ IR and define $M_{LN} := (m + \Delta)_{LN} = [\ln(m+\Delta) - \mu]/\sigma$.

$$\begin{aligned} E[V^-(m)^2] &= (m - Y + \Delta)^2[1 - \Phi(Y_N)] \\ &+ [(\mu - m - \Delta)^2 + \sigma^2][\Phi(Y_N) - \Phi(X_N)] \\ &+ \sigma[2(m + \Delta) - \mu - Y]\varphi(Y_N) \\ &- \sigma[2(m + \Delta) - \mu - X]\varphi(X_N) \\ &+ (m - X + \Delta)^2\Phi(X_N) \ . \end{aligned} \tag{4.7}$$

For the expected value we obtain:

$$\begin{aligned} E(V) &= X\Phi(X_{LN}) + e^{\mu+\frac{\sigma^2}{2}}[\Phi(Y_{LN} - \sigma) - \Phi(X_{LN} - \sigma)] \\ &+ Y[1 - \Phi(Y_{LN})] - \Delta \ . \end{aligned} \tag{4.8}$$

For the variance we obtain:

$$\begin{aligned} \mathrm{Var}(V) &= X^2\Phi(X_{LN}) + Y^2[1 - \Phi(Y_{LN})] \\ &+ e^{2(\mu+\sigma^2)}[\Phi(Y_{LN} - 2\sigma) - \Phi(X_{LN} - 2\sigma)] \\ &- \{X\Phi(X_{LN}) + e^{\mu+\frac{\sigma^2}{2}}[\Phi(Y_{LN} - \sigma) - \Phi(X_{LN} - \sigma)] \\ &+ Y[1 - \Phi(Y_{LN})]\}^2 \ . \end{aligned} \tag{4.9}$$

In the case of $X - \Delta < m \leq Y - \Delta$ we obtain for the shortfall-probability:

$$P(V^-(m) > 0) = \Phi(M_{LN}) \ . \tag{4.10}$$

For the shortfall-expectation we obtain in this case:

Finally we obtain for the shortfall-semivariance in this case:

$$\begin{aligned} \mathrm{E}[V^{-}(m)] &= (m+\Delta)\,\Phi(M_{LN}) - X\,\Phi(X_{LN}) \\ &\quad - e^{\mu+\frac{\sigma^2}{2}}\left[\Phi(M_{LN}-\sigma) - \Phi(X_{LN}-\sigma)\right]. \end{aligned} \tag{4.11}$$

$$\begin{aligned} \mathrm{E}[V^{-}(m)^2] &= (m-X+\Delta)^2\,\Phi(X_{LN}) \\ &\quad + (m+\Delta)^2\left[\Phi(M_{LN}) - \Phi(X_{LN})\right] \\ &\quad - 2(m+\Delta)\,\mathrm{e}^{\mu+\frac{\sigma^2}{2}}\left[\Phi(M_{LN}-\sigma) - \Phi(X_{LN}-\sigma)\right] \\ &\quad + \mathrm{e}^{2(\mu+\sigma^2)}\left[\Phi(M_{LN}-2\sigma) - \Phi(X_{LN}-2\sigma)\right]. \end{aligned} \tag{4.12}$$

Finally in case $m > Y - \Delta$ we obtain for the shortfall-expectation

$$\begin{aligned} \mathrm{E}[V^{-}(m)] &= m - X\,\Phi(X_{LN}) \\ &\quad - e^{\mu+\frac{\sigma^2}{2}}\left[\Phi(Y_{LN}-\sigma) - \Phi(X_{LN}-\sigma)\right] \\ &\quad - Y\left[1-\Phi(Y_{LN})\right] + \Delta. \end{aligned} \tag{4.13}$$

For the shortfall-semivariance we obtain in this case:

$$
\begin{aligned}
E[V^-(m)^2] &= (m - Y + \Delta)^2\,[1 - \Phi(Y_{LN})] \\
&+ (m + \Delta)^2\,[\Phi(Y_{LN}) - \Phi(X_{LN})] \\
&- 2(m + \Delta)\,e^{\mu + \frac{\sigma^2}{2}}\,[\Phi(Y_{LN} - \sigma) - \Phi(X_{LN} - \sigma)] \\
&+ e^{2(\mu + \sigma^2)}\,[\Phi(Y_{LN} - 2\sigma) - \Phi(X_{LN} - 2\sigma)] \\
&+ (m - X + \Delta)^2\,\Phi(X_{LN}) \,.
\end{aligned}
\tag{4.14}
$$

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