AN ACTUARIAL APPROACH TO RISK MANAGEMENT WITH CAT INSURANCE CONTRACTS

PETER ALBRECHT
INSTITUTE FOR INSURANCE SCIENCES
UNIVERSITY OF MANNHEIM
D - 68131 MANNHEIM, SCHLOß
GERMANY

TELEPHONE : +49 621/292-5494
FAX : +49 621/292-5712
E-MAIL : RISK@BWL.UNI-MANNHEIM.DE

Abstract

The present paper gives an actuarial analysis of the applications of CAT insurance contracts (CAT Futures, CAT Options, CAT Call Option Spreads) to the risk management (control of the loss ratio) of insurance companies. We perform a quantitative analysis of the basic positions of CAT insurance contracts as well as of the combined positions of an insurance company using CAT insurance contracts in its management of catastrophe risk.

Keywords

catastrophe insurance futures and options ; risk management ; loss ratio control

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Résumé

L'étude présente contient une analyse actuarielle de l'application de contrats à terme sur l'assurance des risques de catastrophes (CAT-Futures, CAT-Options, CAT-Call Option-Spreads) dans le cadre de la gestion du risque (contrôle du rapport des sinistres aux primes) des compagnies d'assurance. Nous analysons d'une manière quantitative les positions de base des contrats à terme sur l'assurance des risques de catastrophes aussi bien que les positions combinées d'une compagnie d'assurance utilisant ces contrats pour la gestion du risque des catastrophes.

1. Introduction

The Chicago Board of Trade (CBoT) has developed a number of catastrophe insurance futures and option contracts. The trading of these contracts makes it possible for the first time to transfer catastrophe risks on the basis of financial instruments, in contrast to the traditional actuarial instruments in the form of reinsurance contracts. It is the goal of the present paper to give a systematic analysis of the basic positions of CAT insurance contracts as well as of the combined positions of an insurance company using CAT insurance contracts in its management of catastrophe risk. This is done on the basis of a risk theoretical approach, i.e. by giving a stochastic specification of the underlying claims process.

2. Selected Specifications of CAT Insurance Contracts which are Relevant for Model Building

General references for contract specifications are CBoT (1994) and SMITH / PICKLES (1994). In this chapter only those specifications which are used in the risk theoretical model of the claims process are shortly reviewed.
The underlying object of the CAT futures contract is a representative claims index, giving the development of the loss ratio (ratio of aggregated claims and aggregated premiums) of a representative pool of insurance contracts (index collective) and of a certain category of insurance lines subject to catastrophic losses. The index collective emerges from a pooling of the respective risk collectives of selected insurance companies (index

companies). CAT futures typically are defined relative to a specific *loss quarter*, which is defining the reference period for the loss ratio to be calculated. However, because of the "IBNR-problem" the information on the reported and settled claims at the end of the quarter is not identical to the final value of settled claims having occured in this quarter.The pragmatic solution to this problem is as follows: The Final Settlement Value of the CAT futures contract only takes claims into consideration which have occured in the loss quarter and have been reported to the index companies by the end of the subsequent quarter (end of the *reporting period*). The corresponding claim amounts which are used in calculating the claims index are the claim amounts as being paid and reserved at the end of the reporting period. As there is an additional time lag allowed in the reporting of these values by the index companies the Final Settlement Value is made public at the fifth calender day (resp. the subsequent business day) of the seventh month following the end of the loss quarter. This *settlement day* naturally is identical to the end of the *trading period*. Finally it has to be remarked that the premium volume used in the calculation the final loss ratio is an estimated figure and is announced by CBoT prior to the begin of the trading period with respect to a certain loss quarter. This means that the sole source of randomness of the loss ratio is the claims development.

The underlying object of the CAT option contract is the corresponding CAT future. For the buyer of a call option the exercise of the option leads to a long position in the underlying CAT future. For the buyer of a put option the exercise leads to a short position in the future.

3. Quantitative Analysis of Basic Positions

3.1 The Basic Modelling of the Claims Process

As usual in risk theory $\{S(t);\ t \geq 0\}$ denotes the accumulated claims process, i.e. the accumulated claims in the time interval $[0,t]$. Because of the IBNR-problem this process is not observable directly and we consider for every fixed t the stochastic process $\{S(t;t+\tau);\ 0 \leq \tau \leq \omega\}$ which gives at point of time $t+\tau$ the sum of the paid and reserved amounts of the claims which have occured in $[0,t]$. It is assumed that there is a unifom (independent of t, of the insurance line and the insurance company considered) maximal duration ω for the final regulation of all claims having occured in $[0,t]$, especially we have $S(t) = S(t\ ;\ t + \omega)$.

With respect to the remarks made in chapter 2 this conception can be applied as follows. Let $0 < u < v < T$ denote points of time. The time interval $[0,u]$ denotes the loss quarter underlying a specific CAT futures contract, $[0,v]$ the corresponding reporting period and $[0,T]$ the trading period (final settlement of the contract takes place in $t = T$). Let $\pi = \pi\ (u)$ denote the earned premium for the loss quarter $[0,u]$ and $\tilde{\pi}$ the corresponding estimated figure, then the loss ratio announced in $t = T$ (the final loss ratio index value) is

$$LR(v;T) = LR(u;v;T) = \frac{S(u;v)}{\tilde{\pi}}. \tag{1}$$

The final loss ratio index value $LR(u;v;T)$ is related to the claim amounts paid and reserved in $t = v$ for claims having occured in $[0,u]$. The portion of the finally settled claim amount not taken into consideration is

$$\begin{aligned} Z(u;\ v) &= S(u;\ u + \omega) - S(u;\ v) \\ &= S(u;\ u + \omega) - \tilde{\pi}\ LR(u;\ v;\ T). \end{aligned} \tag{2}$$

3.2 Analysis of the CAT Futures Positions

Let denote $LR_l(v;T) = LR_l(u;v;T)$ the final loss ratio index value and $F(T) = F(u;T)$ the final settlement value of a CAT futures contract relating to the loss quarter $[0,u]$, then we have:

$$\begin{aligned} F(T) &= \min\{25.000\ LR_l(v;T),\ 50.000\} \\ &= 25.000\ \min\{LR_l(v;T),\ 2\}. \end{aligned} \tag{3}$$

The final settlement value is 25.000 \$ times the final loss ratio index value, but may not exceed 50.000 \$, i.e. the maximal loss ratio compensated by the futures contract is 2.0.

In case the futures contract was bought in $t = s$ and the futures price at this point of time was $F(s)$, then the final profit (gain / loss) - position $GV(T)$ for the buyer of the contract is

$$GV(T) = F(T) - F(s), \tag{4}$$

the corresponding profit position of the seller of the contract would be $F(s)$ - F(T). If we standardize futures prices per unit of the value of the contract by $F^*(t) = F(t)\ /\ 25.000$, then the corresponding final profit position $GV^*(T)$ would be $\min\{LR_l(v;T), 2\} - F^*(s)$ for the buyer and $F^*(s) - \min\{LR_l(v;T), 2\}$ for the seller.

3.3 Analysis of the CAT Option Positions

Let X denote the exercise price of the option and $V(T) = V(s,u,T)$ the final profit position for the buyer of a call, then we have $V_T = \max\,(F_T - X, 0)$. If we define the corresponding standardized exercise price $LR_X := X\,/\,25.000$ per unit of the value of the contract, which intuitively corresponds to an "exercise loss ratio" then we have from (3):

$$\begin{aligned} V_T &= \max\{25.000\min[LR_I(v;T),2] - 25.000LR_X, 0\} \\ &= 25.000\max\{\min[LR_I(v;T),\ 2] - LR_X,\ 0\}. \end{aligned} \tag{5}$$

In case of $LR_I(v;T) \leq 2$, i.e. the final loss ratio index value is not exceeding 2, as special case we have $V_T = 25.000 \max\,\{LR_I(v;T) - LR_X,\ 0\}$ which intuitively corresponds to the position of the buyer of a call option on a loss ratio. The value of the final call position is positive in case the final loss ratio index value $LR_I(v;T)$ exceeds the "exercise loss ratio" LR_X. In case $LR_I(v;T) > 2$ we have (assuming additionally $LR_X < 2$) as special case $V_T = 25.000\ (2 - LR_X)$, i.e. from a loss ratio value of 2,0 onwards the buyer of the call does not take profit anymore from a higher final loss ratio index value, his profit position is "freezed".

Let denote $C(s)$ the price of the call option in $t = s$ (time of buying the contract) resp. $C^*(s) = C(s)\,/\,25.000$ per unit of the value of the contract. We have as final standardized profit position for the buyer of the call:

$$GV^*(T) = \max\{\min[LR_I(v;T), 2] - LR_X, 0\} - C^*(s). \tag{6}$$

For the buyer of a CAT put option contract with exercise price X the final profit position is $V_T = \max\ (X - F_T, 0)$ and we therefore have from (3) (assuming $LR_X < 2$):

$$\begin{aligned} V_T &= 25.000\ \max\{LR_X - \min[LR_I(v;T), 2], 0\} \\ &= 25.000\ \max\{LR_X - LR_I(v;T),\ 0\}. \end{aligned} \tag{7}$$

Let denote $P(s)$ the price of the put option in $t = s$ (time of buying the contract) and $P^*(s) := P(s)\ /\ 25.000$ the corresponding standardized price. In analogy to (6) we have for the final standardized position:

$$GV^*(T) = \max\{LR_X - LR_I(v;T),\ 0\} - P^*(s). \tag{8}$$

Finally we analyse the position of a call option spread. Let denote X the exercise price of a long call position and $Y > X$ the exercise price of a short call position. The corresponding option premiums are denoted by $C(X)$ resp. $C(Y)$ and from no-arbitrage considerations we have $C(X) > C(Y)$.

The final profit position of a call option spread then is given by:

$$\begin{aligned} \mathbf{GV_T} &= \mathbf{\max(F_T - X,\ 0) - C(X) - [\max(F_T - Y,\ 0) - C(Y)]} \\ &= \mathbf{\min\{\max(F_T - X,\ 0),\ Y - X\} - [C(X) - C(Y)]}. \end{aligned} \tag{9}$$

Let GV^*_T, LR_X, LR_Y, $C^*(X)$ and $C^*(Y)$ denote the corresponding standardized quantities, then we have from (3):

$$GV_T^* = \min\{\max[\min(LR_I(v;T),\ 2) - LR_X,\ 0],$$
$$LR_Y - LR_X\} - [C^*(X) - C^*(Y)]. \tag{10}$$

In case of $LR_I(v;T) \leq 2$ we get as a special case $GV^*_T = \min\{\max[LR_I(v;T) - LR_X, 0], LR_Y - LR_X\} - [C^*(X) - C^*(Y)]$. In case of $LR_I(v;T) \geq 2$ we have under the assumption $LR_X < LR_Y < 2$ the relation $\min\{\max(2 - LR_X, 0), LR_Y - LR_X\} = \min\{2 - LR_X, LR_Y - LR_X\} = LR_Y - LR_X$ and therefore $GV^*_T = LR_Y - LR_X - [C^*_s(X) - C^*_s(Y)]$, i.e. in case of $LR_I(v;T) \geq 2$ the final profit position is constant and identical to the difference of the (standardized) net exercise prices and the net option premiums. This, however, is identical to the profit position in the first case, when we additionally assume $LR_I(v;T) \geq LR_Y$ and therefore the entire position reduces to ($\Delta C^*_s := C^*_s(X) - C^*_s(Y)$, $\Delta LR := LR_Y - LR_X$):

$$GV_T^* = \min\{\max[LR_I(v;T) - LR_X,\ 0],\ \Delta LR\} - \Delta C_s^*$$
$$= \begin{cases} -\Delta C_s^* & LR_I(v;T) \leq LR_X \\ LR_I(v;T) - LR_X - \Delta C_s^* & LR_X \leq LR_I(v;T) \leq LR_Y \\ \Delta LR - \Delta C_s^* & LR_I(v;T) \geq LR_Y. \end{cases} \tag{11}$$

4. Quantitative Analysis of the Risk Management Positions

4.1 Preliminary Remarks

First of all we have to take into consideration that the final loss ratio index value $LR_I(v;T)$ is defined by the accumulated claims process of the *index collective*. Therefore the first category of modelling assumptions is concerned with the specification of the functional relation between the final loss ratio index value $LR_I(v;T)$ on one hand and the final loss ratio $LR(v;T)$ of the respective collective of the considered insurance company, which uses CAT insurance contracts in its management of catastrophe risk on the other.

In analogy to 3.1 therefore let $\{S(t)\}$ resp. $\{S(t;t+\tau)\}$ denote the corresponding accumulated claims processes for the considered collective of insured risks. Let $\pi = \pi(u)$ denote the corresponding earned premium for the loss quarter $[0,u]$ for that collective.

Let in addition denote $LR(v;T) = LR(u;v;T) = S(u;v) / \pi$ the loss ratio of the fixed collective of risks for claims that have occured in $[0,u]$ but only using the information (paid and reserved claim amounts) available in $t = v$. The calculation of this number as well as the calculation of the corresponding profit value shall take place in $t = T$. Clearly in $t = T$ the information available is the figure $S(u;T)$, however, when analysing the effects of using CAT insurance contracts only the information available in $t = v$ is relevant. This is because of $[0,v]$ and not $[0,T]$ is the reporting period underlying the CAT contracts. Only with this at the first glance artificial construction it is guaranteed that the correctly comparable positions are used, when studying the effects of using CAT insurance contracts in risk management.
As the central hypothesis on the link between $LR_I(v;T)$ and $LR(v;T)$ we postulate ($\beta \neq 0$)

$$LR(v;T) \equiv \alpha + \beta \; LR_I(v;T) \tag{12a}$$

resp. as a special case

$$LR(v;T) \equiv LR_I(v;T). \tag{12b}$$

In case of (12b) the individual loss ratio (as evaluated in $t = T$ based on the information available in $t = v$) of the fixed collective of risks insured and the final loss ratio index value are completely identical, i.e. there is no *cross-hedge risk*. In case of (12a) a *strictly* linear relation between the two loss ratios is postulated. A more general hypothesis would be, that a certain functional link of the two loss ratios is superimposed by a noise term with expected value of zero.

In case it is not $S(u;v)$ but $S(u) = S(u;u+\omega)$ that we want to control, this can easily be done if a second category modelling assumptions concerning the link of these two figures is postulated. In the following analysis we will concentrate on a simple hypothesis of the form ($\gamma = \gamma(v)$):

$$S(u;v) \equiv \gamma\ S(u), \tag{13}$$

i.e. we assume that until $t = v$ always a fixed fraction $0 < \gamma < 1$ of the final accumulated claim amount for claims in $[0,u]$ is known (paid and reserved).

Herewith the hypotheses underlying our analysis are completely specified. The hypotheses chosen always presuppose that the CAT insurance contract is hold until maturity of the contract, i.e. until the end of the trading period. Clearly a cancellation of the CAT contract at every time prior to maturity via cash settlement at market prices is possible (this is a central distinguishing feature of buying/selling contracts at a futures/options exchange), however, then the investor has to take *basis risk* (caused by the possibility of an non-synchronized development of $\{S(t;t+\tau)\}$ and $\{S_I(t;t+\tau)\}$) into consideration additionally. An *ex ante*-analysis of basis risk, however, makes it necessary to specify a hypothesis of the link between these two stochastic processes. This proves to be very problematic, as the claims development of the index pool of risks cannot be observed exogeneously and, in addition even ex post, i.e. for already settled CAT contracts, there is no infomation available with respect to this.

4.2 Hedging with CAT Insurance Futures

Central to our analysis is the *technical* profit (gain/loss) position TGV of the insurance company for the loss quarter $[0, u]$ before the purchase of CAT futures on one hand and after the purchase on the other. The technical profit position is calculated in $t=T$ and is only taking the claims infomation (paid and reserved claims) available in $t=v$ into consideration. Before the purchase of CAT futures we have

$$TGV(v;T) = TGV(u;v;T) = \pi(u) - S(u,v). \tag{14}$$

Using the insurance company's loss ratio $LR(v; T)$ as defined in 4.1 we obtain

$$TGV(v;T) = \pi - \pi\ LR(v;T) = \pi\ [1 - LR(v;T)]. \tag{15}$$

The technical profit position $TGV(v; T)$ is not identical to the *final* technical profit position for the loss quarter $[0, u]$, which would be

$$TGV(u;T) = \pi(u) - S(u) = \pi(u) - S(u; u+\omega). \tag{16}$$

The amount not taken into consideration when working with $TGV(v; T)$ is

$$\Delta_{TGV}(v;T) = \pi - S(u;v) - [\pi - S(u;u+\omega)]$$
$$= S(u;u+\omega) - S(u;v). \tag{17}$$

Let denote

$$LR(u;T) := \frac{S(u)}{\pi} \tag{18}$$

the *final loss ratio* for the loss quarter $[0, u]$, then we have

$$\Delta_{TGV}(v;T) = \pi\,[LR(u;T) - LR(u;v;T)]. \tag{19}$$

Let us now consider the hedge position. As the original position of the insurance company is a *short position* (claims are paid) the relevant hedge position is the *long hedge*, i.e. the company has to *buy* CAT futures. In case of a 1:1 hedge the number of contracts to be bought is $x_\pi := \pi/25000$, i.e. the premium value of the collective of risks to be hedged is divided by the value of a CAT futures contract. The final technical profit position $TGV^H(v; T) = TGV^H(u; v; T)$ of the company *after* realizing the hedge operation (holding the futures to maturity) then is given by (using (3)):

$$TGV^H(v;T) = \pi - S(u;v) + x_\pi 25000 \min\{LR_I(v;T), 2\} - x_\pi F_s$$
$$= \pi\,[1 - LR(v;T) + \min\{LR_I(v;T), 2\} - F_s^*]. \tag{20}$$

In this expression F_s denotes the price of the futures contract in $t=s$, when the futures are bought and the hedge position is established. $F_s^* := F_s/25000$ denotes the corresponding standardized position. We continue the analysis in "loss ratio terms" and define a *hedge-loss ratio* by

$$LR^H(v;T) = LR^H(u;v;T) = 1 - \frac{TGV^H(v;T)}{\pi}, \tag{21}$$

i.e. we have $TGV^H(v; T) = \pi\,[1-LR^H(v; T)]$. From (20) we then obtain:

$$LR^{H}(v;T) = LR(v;T) - \min\{LR_I(v;T), 2\} + F_s^*. \tag{22}$$

This means that central for the effects of the perfomed hedge operation is the link betweeen the loss ratio of the index collective of risks and the insurance company's collective of risks. The ideal case would be that there is no cross-hedge risk at all, i.e. assumption (12 b) would be true. Using this assumption and looking at the special case $LR(v;\ T) = LR_I(v;\ T) \leq 2$ we finally obtain $LR^H(v;\ T) = F_s$.

As F_s is a known figure, the total position reduces to a riskless position, i.e. we have a *perfect hedge* position. The corresponding technical profit position is $TGV^H(v;\ T) = \pi\ [1\text{-}F_s^*]$, i.e. the insurance company earns a riskless profit in case $F_s^*<1$ resp. is realizing a loss of an ex ante known amount in case $F_s^*>1$. The realisation of a perfect hedge has the effect of locking in the hedge-loss ratio at an ex ante known amount independent of the realized original loss ratio. However, in case of insurance futures the perfect hedge position cannot be realized completely even when not allowing for cross-hedge risk. In case of $LR(v;\ T) = LR_I(v;\ T) > 2$ we obtain $LR^H(v;\ T) = LR(v;\ T)\text{-}2+F_s^*$. This means that the restriction of the loss ratio to the value of 2,0 for a CAT futures contract leads to an incomplete perfect hedge position.

The total position of the 1:1 hedge is given by:

$$LR^{H}(v;T) = \max\{LR(v;T) - 2,\ 0\} + F_s^* = \begin{cases} F_s^* & LR(v;T) \le 2 \\ LR(v;T) - 2 + F_s^* & LR(v;T) > 2\ . \end{cases} \tag{23}$$

This means that in the case of non-existence of cross hedge-risk, the realization of a perfect hedge position is endangered

- in case the loss ratio of the insurance company exeeds the amount of 2,0 or
- in case the CAT futures contract is cancelled prior to maturity (basis risk).

In the next step of the analysis we still assume (12 b), i.e. there is no cross hedge risk, but now we are investigating the effects of varying the hedge ratio. The total value $\tilde{\pi}$ of the CAT futures contracts bought is assumed to be a multiple of the premium volume of the hedged collective of risks, i.e. $\tilde{\pi} = k\pi$ resp. $\tilde{x}_\pi = k x_\pi$. The resulting technical profit position after hedge then is given by

$$TGV^{H}(v;T) = \pi\,[1 - LR(v;T) + k\min\{LR_I(v;T), 2\} - kF_s^*], \tag{24}$$

the corresponding hedge-loss ratio by

$$LR^{H}(v;T) = LR(v;T) - k\min\{LR_I(v;T), 2\} + k\ F_s^* \tag{25}$$

and therefore we obtain from (12 b)

This means e.g. in case of $LR(v;\ T) \le 2$ that a reduction of the extent of the hedge (k<1) leads to a corresponding reduction - neglecting the additive term

$$LR^{H}(v;T) = \begin{cases} (1-k)LR(v;T) + kF_s^* & LR(v;T) \le 2 \\ LR(v;T) - 2k + kF_s^* & LR(v;T) \ge 2. \end{cases} \tag{26}$$

kF_s^* - of the loss ratio by a factor of (1 - k). This relative reduction of the loss ratio can be a proper goal of a hedge operation with CAT futures itself. On the other hand, an increase of the extent of the hedge (k > 1) can be a proper goal itself, too, especially when we take into consideration that $LR(v; T)$ is not identical to the final loss ratio $LR(u;T)$ according to (18) and it would be the intention of the insurance company to additionally cover the difference according to (19) by the hedge operation.

The corresponding analysis can be performed in a simple manner if we are willing to assume hypothesis (13), i.e. that at time $t=v$ always a fixed fraction γ of the final accumulated claim amount for claims in $[0,u]$ is known (paid and reserved). Using the final technical profit position $TGV(u;T)$ according to (16) and the final loss ratio $LR(u;T)$ according to (18) the analysis is performed as follows. From (13) we have LR(u; T) = LR(v; T) / γ and therefore $TGV(u;T) = \pi\left[1 - \frac{1}{\gamma} LR(v;T)\right]$. The hedge operation is performed on the basis of a hedge ratio of k, i.e. the number of CAT futures bought is $\tilde{x}_\pi = k\, x_\pi$. The resulting technical profit position after hedge is given by

$$TGV^{H}(u;T) = \pi[1 - \frac{1}{\gamma}LR(v;T) + k\min\{LR_f(v;T), 2\} - kF_s^*], \tag{27}$$

and the corresponding loss ratio is given by

$$LR^{H}(u;T) = \frac{1}{\gamma} LR(v;T) - k \min\{LR_{l}(v;T), 2\} + kF_s^{*}. \tag{28}$$

Using hypothesis (12 b) this expression reduces to

$$LR^{H}(u;T) = \frac{1}{\gamma} LR(v;T) - k \min\{LR(v;T), 2\} + kF_s^{*}$$

$$= \begin{cases} \left(\frac{1}{\gamma} - k\right) LR(v;T) + kF_s^{*} & LR(v;T) \le 2 \\ \frac{1}{\gamma} LR(v;T) - 2k + kF_s^{*} & LR(v;T) \ge 2. \end{cases} \tag{29}$$

This means when choosing the specific ratio $k = \frac{1}{\gamma} > 1$ the realisation of a riskless position is possible again, at least in case of $LR(v;T) \le 2$. Otherwise a proportional reduction of the loss ratio is possible (neglecting the additive term F_s^{*}).

We now perfom the analysis on the basis of the strictly linear relationship (12 a) as well as assuming a hedge ratio which is not restricted to the value of one. The resulting loss ratio after hedge is given by

$$LR^{H}(v;T) = LR(v;T) - k \min\left\{\frac{1}{\beta}[LR(v;T) - \alpha], 2\right\} + kF_s^*$$

$$= \begin{cases} \left(1 - \frac{k}{\beta}\right) LR(v;T) + \frac{k\alpha}{\beta} + kF_s^* & LR_I(v;T) \leq 2 \\ LR(v;T) - 2k + kF_s^* & LR_I(v;T) > 2. \end{cases} \quad (30)$$

As to be expected, one can realize a perfect hedge position with the amount of $\alpha + \beta F_s^*$ by choosing $k = \beta$, at least as long the loss ratio index value does not exeed the value of 2,0. In the general case choosing the hedge ratio k leads to a proportional change of the loss ratio of the insurance company (neglecting the additive term $\frac{k\alpha}{\beta} + kF_s^*$).

4.3 Capital Protection with CAT Insurance Options: The Call Hedge

Keeping up the approach and nomenclature as before, we obtain on the basis of expression (6) the following technical profit position after the hedge operation, which consists of buying $k\pi/25.000$ CAT insurance option contracts :

$$TGV^{H}(v;T) = \pi[1 - LR(v;T) + k\max\{\min[LR_I(v;T),2] - LR_X, 0\} - kC_s^*], \quad (31)$$

resp. we obtain in loss ratio terms

$$LR^{H}(v;T) = LR(v;T) - k\max\{\min[LR_I(v;T), 2] - LR_X, 0\} + kC_s^*. \quad (32)$$

Once again we have to specify the link between $LR(v;T)$ and $LR_I(v;T)$ if we want to evaluate (32) further. We first concentrate on the hypothesis (12b), i.e. the non-existence of cross hedge risk and look at the case $k = 1$, i.e. the case of a 1:1 call hedge.

In case $LR_I(v;T) \leq 2$ (32) is reduced to the position

$$LR^H(v;T) = \min \{LR(v;T),\ LR_X\} + C_s^* = \begin{cases} LR_X + C_s^* & LR(v;T) \geq LR_X \\ LR(v;T) + C_s^* & LR(v;T) \leq LR_X . \end{cases} \qquad (33)$$

The 1:1 call hedge in this situation has the effect of *limiting* the hedge loss ratio of the insurance company to the amount of $LR_X + C_s^*$ in the case of an adverse development of the original loss ratio, i.e. when exceeding the exercise loss ratio LR_X. In the other case the hedge loss ratio is identical to the original loss ratio, but increased by the costs of the hedge operation in form of the option premium. The effects of a CAT insurance call hedge so far are very similar to a stop loss reinsurance contract with an unlimited layer.

However, we in addition have to take into consideration the possibility of $LR_I\ (v;T) \geq 2$. In this case we obtain (under the assumption $LR_X < 2$) $LR^H(v;T) = [LR(v;T) - 2] + LR_X + C_s^*$, i.e. the hedge-loss ratio is not limited any more but increases proportional to the original loss ratio reduced by the amount $2 - LR_X - C_s^*$. The total position in case of $LR\ (v;T) = LR_I(v;T)$ and $k = 1$ therefore is given by

Finally we look at the general position, i.e. we allow for the strictly linear

$$LR^H(v;T) = \min\{LR(v;T), \max[LR_X, LR_X + LR(v;T) - 2]\} + C_s^*$$

$$= \begin{cases} \min\{LR(v;T), LR_X\} + C_s^* & LR_I(v;T) \le 2 \\ [LR(v;T) - 2] + LR_X + C_s^* & LR_I(v;T) \ge 2. \end{cases} \qquad (34)$$

relationship (12a) and for a general hedge ratio k. In this case (32) reduces to:

$$LR^H(v;T) = LR(v;T) - k \max\left\{\min\left[\frac{1}{\beta}[LR(v;T) - \alpha], 2\right] - LR_X, 0\right\} + kC_s^*. \qquad (35)$$

In the hedge-area $LR_X \le LR_I\ (v;T) \le 2$ this position further reduces to:

$$LR^H(v;T) = \left[1 - \frac{k}{\beta}\right] LR(v;T) + k\left[\frac{\alpha}{\beta} + LR_X + C_s^*\right]. \qquad (36)$$

This means especially that when choosing the hedge ratio $k = \beta$ the insurance company can limit the loss ratio to an amount of $\alpha + \beta\ LR_X + \beta\ C_s^*$ within the hedge area.

An alternative goal of the hedge operation could be to control the final loss ratio $LR(u;T)$ according to (18). As hypothesis we postulate the relations (12b) and (13). In this special case the hedge-loss ratio is given by ($LR(u;T) \equiv LR(v;T)/\gamma$):

$$LR^H(u;T) = \frac{1}{\gamma} LR(v;T) - k \max\{\min[LR(v;T), 2] - LR_X, 0\} + kC_s^*. \qquad (37)$$

In the area $LR(v;T) \equiv LR_I\ (v;T) \le 2$ this expression further reduces to:
This means especially that when choosing $k = 1/\gamma$ we obtain the hedge-loss ratio $LR^H(u;T) =$ [min $\{LR\ (v;T), LR_X\} + C_s^*] / \gamma$ which is identical to $1/\gamma$ times the hedge position (33).

$$LR^{H}(u;T) = \begin{cases} \left(\frac{1}{\gamma} - k\right) LR(v;T) + k(LR_X + C_s^*) & LR(v;T) \geq LR_X \\ \frac{1}{\gamma} LR(v;T) + k\,C_s^* & LR(v;T) \leq LR_X . \end{cases} \tag{38}$$

4.4 The Covered Short Put Position

The covered short put position of an insurance company using CAT insurance options in its management of catastrophic risk is constructed in analogy to the covered short call position in traditional financial risk management. The insurance company acts as a seller of CAT insurance puts. Concentrating on hypothesis (12b) (non-existence of cross hedge risk) and the case of a 1:1 position (total value of sold puts is identical to the premium volume π) we obtain the following final technical profit position after the short put operation from (8):

$$TGV^{CSP}(v;T) = \pi[1 - LR(v;T) - \max\{LR_X - LR(v;T), 0\} + P_s^*], \tag{39}$$

resp. in loss ratio terms

$$\begin{aligned} LR^{CSP}(v;T) &= LR(v;T) + \max\{LR_X - LR(v;T), 0\} - P_s^* \\ &= \max\{LR_X, LR(v;T)\} - P_s^* \\ &= \begin{cases} LR_X - P_s^* & LR(v;T) \leq LR_X \\ LR(v;T) - P_s^* & LR(v;T) \geq LR_X . \end{cases} \end{aligned} \tag{40}$$

From this position we see that in the area $LR(v;T) > LR_X - P_s^*$ the hedge-loss ratio is *lower* than the original loss ratio. However, we have no hedge effect in the narrow sense of limiting the *absolute* amount of the hedge-loss ratio. In the area $LR(v;T) \geq LR_X$ we only have a reduction of the loss ratio by the amount of the option premium received. On the other hand, in the area $LR(v;T) \leq LR_X - P_s^*$ the insurance company cannot take profit of a favourable development of the original loss ratio, because the buyer of the option will exercise his right in this case. Generally this means that the positive effects of the covered short put are only valid in a rather narrow area around LR_X and the insurance company needs to have a very good control of the original loss ratio $LR(v;T)$ to take systematic benefits from the covered short put position.

4.5 Hedging with CAT Insurance Call Option Spreads

In case the insurance company is buying $k\pi/25.000$ CAT call option spread contracts then we have from (11) the following final technical profit position ($\Delta C_s^* := C_s^*(Y) - C_s^*(X)$, $\Delta LR := LR_Y - LR_X$) :

$$TGV^H(v;T) = \pi\,[1 - LR(v;T) + k\min\{\max[LR_I(v;T) - LR_X, 0], \Delta LR\} - k\,\Delta C_s^*], \tag{41}$$

resp. in loss ratio terms:

$$LR^H(v;T) = LR(v;T) - k\min\{\max[LR_I(v;T) - LR_X, 0], \Delta LR\} + k\Delta C_s^*. \tag{42}$$

Once again we have to specify the link between $LR(v;T)$ and $LR_I(v;T)$ to be able to evaluate this position further. Concentrating on hypothesis (12b) and on the case $k=1$ we obtain:

$$LR^{H}(v;T) = \min\{LR(v;T), \max[LR_X, LR(v;T) - \Delta LR]\} + \Delta C_s^*$$

$$= \begin{cases} LR(v;T) + \Delta C_s^* & LR(v;T) \leq LR_X \\ LR_X + \Delta C_s^* & LR_X \leq LR(v;T) \leq LR_Y \\ LR(v;T) - \Delta LR + \Delta C_s^* & LR(v;T) \geq LR_Y. \end{cases} \tag{43}$$

In comparison to the corresponding position (34) we notice that an absolute limitation of the hedge-loss ratio in only possible in the interval $[LR_X, LR_Y]$. The effects of a CAT insurance call option spread are very similar to a stop loss reinsurance contract with a limited layer.

5. Concluding Remarks

The present paper gives an actuarial analysis of the effects when applying CAT insurance futures and option contracts in the management of catastrophe risk of an insurance company. The hypotheses necessary for the analysis have been specified in 4.1. They relate to the extent of the cross hedge risk, the link between the claim amounts $S(u;\, v)$ and $S(u)$ according to expression (13) and the assumption that all CAT contracts are hold until maturity (non-existence of basis risk).

References

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