

# Allocating Capital Using the Expected Value of Default

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## Abstract

This paper develops a new method to allocating capital. First the expected value of default is allocated based on the economic value of payoff to a policy, counting the cost of insolvency. Then the capital is allocated accordingly so that a policy's contribution to the default value equals its allocated default value. Only part of the capital is allocated to the policies, the rest is attributed to the risky assets. The allocated capitals are given in a simple, closed-form mathematical formula. The derivation of the formula is straightforward and is economically sound. A real world example illustrates the step-by-step calculation, which can be easily implemented in a spreadsheet.

**Keywords.** Cost of capital, capital allocation, expected value of default.

## 1 INTRODUCTION

Many methods have been developed over the years to allocating capital in an insurance company. Most of them may be labeled “operational”, meaning that they provide a multi-step procedure instead of a closed-form mathematical formula. A typical application begins with the selection of a risk measure. The risk measure is used to compute a marginal capital for each line of business. The company total capital is then allocated in proportion to the marginal capitals. See Venter [7] for a survey of the methods. The lacking of a closed-form formula is a drawback. It hinders the further study of the allocated capitals. Little can be said about their relative size before the last step is finished. It is difficult to see how a line’s risk profile contributes to the company risk and affects the allocation. Furthermore, all the methods contain some arbitrary procedures that make little economic sense. (One such procedure is allocating capital “in proportion to” the marginals.) They are relied upon because of convenience and the absence of better alternatives. As of today, the capital allocation is still a much debated topic without a satisfactory solution.

A company holds a certain level of capital to keep it from insolvent. So the expected value of default, sometimes viewed as the price of a put option on the company asset, is a sensible risk measure for determining and allocating capital. In fact, when the cost of insolvency is considered, the premium calculation requires an evaluation of the default value and a proper allocation of the default value to the policyholders. This idea was first explored in Phillips, Cummins and Allen [5], and in Myers and Read [4]. The latter paper becomes very influential among the actuaries. If the capital is determined by a target default ratio, the ratio of the expected value of default to the liability, then a small change in a line’s exposure causes a small change in capital. A key observation in Myers and Read [4] is that these “marginal capitals” for all lines add up to the total company capital. So one can set the capital allocated to a line equal to this marginal capital. The paper then proposes an option pricing approach to calculating the marginal capitals. (Butsic [1] also contributes to this method.) Although original and inspiring, the method suffers from several flaws. Conceptually the marginal capitals are not appropriate candidates for the allocated capitals since the insured losses are not “homogeneous” (Mildenhall [3]).

Technically, the option pricing theory is not readily applicable in insurance. The following assumptions are required by the theory but are unconfirmed in the real insurance world: the asset and the liability are joint lognormal; and the default value can be dynamically replicated by the asset and the liability.

Our approach in this paper also begins with the expected value of default but takes a different path. The first step is to allocate the default value by line of business (or by policy). Sherris [6] found a simple solution based on the economic value of payoff by line, counting the possibility of insolvency. It appears to be the only economically sound solution to allocating the default value, at least for pricing purpose. (Phillips *et al.* [5] and Myers and Read [4] use a different allocation scheme that keeps a constant default ratio across all lines. This is inconsistent with the economic value of payoffs.) The next step is to split the capital by line to match this allocation of default value. We show that, for a given split of capital, there is a natural way to define each line's contribution to the company default value. Only one split has the property that all lines's contribution to default value matches the above allocated default value, which is our solution.

Capital allocation is used in many decision-making processes. Our method is developed for the calculation of cost of capital in pricing. The company risk management also calls for capital allocation. One example is to rank the business units by the ROE, the ratio of operating profit to the allocated capital. Our allocation may apply to this task as well.

The paper is organized as follows. In Section 2 we discuss the capital allocation problem in pricing. The premium calculation requires consistent allocations of the capital and the expected value of default. Section 3 explains the allocation of default value derived in Sherris [6]. This allocation is unique and economically sound. Section 4 shows that, when the company holds risky investments, part of the capital should be allocated to assets. It would be unfair to attribute all capital to the policyholders and charge the cost of capital accordingly. So the total capital is split into the asset and the liability components. In Section 5 we derive the formula to allocate the liability component of capital to the policyholders. The derivation is straight-forward. No option theory is used. The formula is given in a closed-form and is distribution-free. The size of the allocated capital de-

pends on the joint distribution of the liabilities and the assets in the event of default. Section 6 illustrates the step-by-step calculation using a real world example. The algorithm can be easily implemented on a spreadsheet, with the help of a simulation tool like @Risk. In the concluding remarks we comment on the calculation of the ROE.

## 2 RISK COMPONENTS IN PREMIUM

The policy premium covers expected losses and expected underwriting, claim adjusting and other expenses. Since the insurance companies are risk averse, the premium also includes a provision for risk. The risk provision may be conveniently split up into two components. The market risk load accounts for the uncertainty in the policy loss and the economic conditions on the insurance market. The cost of capital includes various costs related a company's holding capital. The premium is then a sum of the following components

$$\begin{aligned} \text{premium} = & \text{expected loss} + \text{market risk load} \\ & + \text{cost of capital} - \text{expected default} + \text{expenses}. \end{aligned} \tag{2.1}$$

When a company defaults, a portion of the outstanding claims will not be paid. To be fair to the policyholders, the expected value of the default amount should be subtracted out from the premium. (More on the expected default in Section 3.) The risk components have been studied extensively in recent years. A survey of actuarial and finance literature on the topic can be found in a CAS research committee report (Cummins *et al.* [2]).

A complete ratemaking process should address all items in (2.1). Sometimes in pricing primary layer coverages one or more of the risk items are considered insignificant, thus are not explicitly evaluated. Instead a contingency load is judgementally selected to provide a cushion for risk. The risk components become significant in pricing large and complex contingency transactions, e.g., excess layer, reinsurance, loss portfolio transfer, or acquisition of a company. A sound theory and accurate calculation methods are needed.

The expected loss plus the market risk load is called the market value (or market price) of loss. In a competitive and efficient insurance market,

supply and demand determines a unique price. If the losses could be traded, the market value would represent a fair price the policyholder should pay. The market value is unrelated to the particular insurance company from which the policy is issued. The calculation of the market value is outside the scope of this article. Some recent development can be found in Cummins *et al.* [2] and Wang [11]. In this article we assume the existance of the market value and develop a theory on the risks related to an insurance company.

The (frictional) cost of capital exists simply because a company holds capital. It is a cost to the company shareholders and is eventually charged to the policyholders through premium. For a US company the largest cost maybe taxes on the investment income of capital, due to double-taxation. Other examples of the cost of capital include the agency cost and the loss of investment opportunity. Venter [8] has a thorough discussion on all types of costs. It is usually assumed that the cost of capital is in direct proportion to the capital amount allocated to a policy. Thus the calculation of the cost of capital is essentially the allocation of capital. Capital allocation also plays a role in company risk management. The operating efficiency of a line of business is usually measured by ROE, which is the operating income (return) divided by the allocated capital (equity).

Since in this paper we focus on the risk items, expenses are left out the equation. Let  $p_i$  be a policy premium net of expenses,  $l_i$  the market value of policy loss,  $d_i$  the expected default, and  $k_i$  the amount of capital allocated to the policy. Equation (2.1) can be written as

$$p_i = l_i - d_i + t \cdot k_i, \quad (2.2)$$

where  $t$  is a constant across all policies in the company. This is the equation studied in Myers and Read [4] and Butsic [1]. In Butsic [1]  $t$  is a function of the income tax rate, but it can also include other factors. All variables in (2.2) should be considered present values at the time of policy issuance. In the next two sections we calculate  $d_i$  and  $k_i$ .

### 3 ALLOCATING EXPECTED VALUE OF DEFAULT

An insurance company defaults when its total liability becomes greater than the total asset. The difference between the liability and the asset is called

the value of default. What happens after a company defaults depends on the regulation. In an unregulated insurance market, the policyholders of the company bear the entire cost. A total loss amount equal to the value of default would not be recovered. Therefore, the expected value of default, evaluated at the policy issuance, should be subtracted out from the total company premium. Correspondingly for an individual policy, the expected unrecoverable loss to the policy should be deducted from its premium. This is the  $d_i$  term in (2.2).

The same argument should also apply to a regulated insurance market, where a guarantee fund is set up by assessing all insurance companies. When one company defaults, the fund takes over the losses the company is unable to pay. So the cost to the policyholders is significantly reduced. In theory, there is still a chance, although very small, that the entire industry would default. Treating the industry like one single insurance company, we can similarly define  $d_i$  to be the expected unrecoverable loss to a policy, and deduct this amount from the premium. However, a real guarantee fund mechanism is far from perfect. The fund incurs its own cost; the policyholders sometimes do not get full recovery from the fund; and mismatch exists between the guarantee fund assessment and the company expected value of default. Because of these difficulties, we limit our discussion to the unregulated market.

Consider a one-period model, where all policies are written at time 0 and all losses are paid at time 1. Assume an insurance company writes  $n$  policies. A policy loss payment at time 1 is a random variable  $L_i$ , whose market value at time 0 is  $l_i$ . Also assume the company holds  $m$  different assets at time 0 with market values  $a_1, \dots, a_m$ . At time 1 they have random values  $A_1, \dots, A_m$ . The company assets and liabilities are summarized below

$$\begin{aligned} \text{total liability at time 0: } l &= \sum_{i=1}^n l_i , \\ \text{total liability at time 1: } L &= \sum_{i=1}^n L_i , \\ \text{total asset at time 0: } a &= \sum_{j=1}^m a_j , \\ \text{total asset at time 1: } A &= \sum_{j=1}^m A_j . \end{aligned}$$

Assume the company is solvent at time 0. Then  $k = a - l > 0$ . This is the company capital at time 0.  $k$  is called the *economic* capital, since  $a$  and

$l$  are market values.

The company defaults at time 1 if  $L > A$ , and the default value is  $D = L - A$ . Denote the default set by  $\Omega = \{L_1, \dots, L_n, A_1, \dots, A_m | L > A\}$ . Assume the risk free rate is  $r$ . The following formula gives the (present value of) expected value of default

$$d = \frac{1}{1+r} E[D 1_\Omega] = \frac{1}{1+r} E[(L - A) 1_\Omega], \quad (3.1)$$

where the function  $1_\Omega$  equals 1 on  $\Omega$  and 0 elsewhere.  $d$  is also called the expected policyholder deficit in actuarial literature. In Myers and Read [4], Butsic [1] and Sherris [6],  $d$  is viewed as the price of an insolvency exchange option. The option theory is not used in this paper.

When the company defaults, the entire asset is paid to the policyholders. Assume the payments are made in such a way that all policyholders get the same percentage of their outstanding losses. So the  $i$ th policyholder is paid an amount of  $L_i/L \cdot A$ , and the remaining  $L_i/L \cdot (L - A)$  is unrecoverable. The expected value  $E[L_i/L \cdot (L - A) 1_\Omega]$  is the expected unrecoverable loss to the policyholder. Its present value

$$d_i = \frac{1}{1+r} E \left[ \frac{L_i}{L} (L - A) 1_\Omega \right]. \quad (3.2)$$

is thus the required  $d_i$  term in (2.2). Since  $d = \sum_{i=1}^n d_i$ , (3.2) provides an allocation of  $d$ .

Formula (3.2) is the only economically sound method to allocating the expected value of default. Any other way would favor some policyholders over others. (3.2) is first explicitly shown in Sherris [6], where a credit is also given to Phillips, Cummins and Allen [5]. A similar idea also appears in Myers and Read [4]. But the argument there leads to an allocation of  $d$  so that all policies have a common default ratio, i.e.,  $d_i/l_i = d/l$ . As pointed out in Sherris [6], such an equation holds only if the losses are deterministic, obviously an unrealistic assumption.

## 4 ALLOCATING CAPITAL TO ASSETS

We develop a method of allocating capital based on formulas (3.1) and (3.2). It is reasonable to allocate part of the capital to the risky assets. It is the

company's decision to take on higher investment risk to generate more profit. The investment results, either gain or loss, only benefit the company shareholders. But the extra risk may increase the expected value of default, resulting in higher capital requirement. If the entire capital were allocated to the policies, then the policyholders would bear additional cost with no benefit, which is unfair. Vrieze and Brehm [9] addresses this important point.

We first split the total capital into two parts,  $k = k^l + k^a$ , presumably the capital amounts allocated to the total liability and the total asset. In the event of insolvency, the value of default is

$$\begin{aligned} D &= L - A = L - (l + k)(1 + r) + a(1 + r) - A \\ &= [L - (l + k^l)(1 + r)] + [(a - k^a)(1 + r) - A]. \end{aligned}$$

Imagine a hypothetical company that carries a loss portfolio  $L$ , holds an initial capital  $k^l$ , and invests its total asset  $l + k^l$  risk-free. Then the value of default is  $L - (l + k^l)(1 + r)$ , whenever  $L > (l + k^l)(1 + r)$ . For our actual company, we may think of  $L - (l + k^l)(1 + r)$  as the value of default contributed by the liabilities, defined on the default set  $\Omega$ . Similarly,  $(a - k^a)(1 + r) - A$  is the value of default contributed by the assets, also defined on  $\Omega$ . The corresponding expected values, discounted to time 0, are  $E[(L/(1+r) - l - k^l)1_\Omega]$  and  $E[(a - k^a - A/(1+r))1_\Omega]$ . By (3.1) we have

$$E \left[ \left( \frac{L}{1+r} - l - k^l \right) 1_\Omega \right] + E \left[ \left( a - k^a - \frac{A}{1+r} \right) 1_\Omega \right] = d.$$

As discussed in Section 3, the entire expected value of default,  $d$ , is allocated to the policies, and none should be allocated to the assets. So we set

$$E \left[ \left( \frac{L}{1+r} - l - k^l \right) 1_\Omega \right] = d, \quad (4.1)$$

$$E \left[ \left( a - k^a - \frac{A}{1+r} \right) 1_\Omega \right] = 0. \quad (4.2)$$

Solving for  $k^l$  and  $k^a$  we have

$$k^l = \frac{1}{\text{Prob}(\Omega)} \left( E \left[ \left( \frac{L}{1+r} - l \right) 1_\Omega \right] - d \right), \quad (4.3)$$

$$k^a = \frac{1}{\text{Prob}(\Omega)} E \left[ \left( a - \frac{A}{1+r} \right) 1_\Omega \right]. \quad (4.4)$$

$k^a$  can be further split into  $m$  components  $k_1^a, \dots, k_m^a$ , so that

$$E \left[ \left( a_j - k_j^a - \frac{A_j}{1+r} \right) 1_\Omega \right] = 0. \quad (4.5)$$

This equation is parallel to (4.2). It has a similar interpretation: the expected default should only be allocated to the policies but not to any of the assets. Solving for  $k_j^a$ , yields

$$k_j^a = \frac{1}{\text{Prob}(\Omega)} E \left[ \left( a_j - \frac{A_j}{1+r} \right) 1_\Omega \right]. \quad (4.6)$$

This is the capital amount allocated to the  $j$ th asset. It is easy to check that  $\sum_{j=1}^m k_j^a = k^a$ . This capital provides a deduction to the value of a risky asset. The  $j$ th asset has a market value of  $a_j$ , but a lower “default-free value” of  $a_j - k_j^a$ . The size of  $k_j^a$  depends on how often and by how much the present value of the asset,  $A_j/(1+r)$ , falls below its market value,  $a_j$ , in the event of default.  $k_j^a$  is not used in ratemaking, but it provides a guidance to the selection of investments.

In particular, if the  $j$ th asset is risk free, then  $A_j = (1+r)a_j$ . (4.6) gives  $k_j^a = 0$ , that is, no capital is allocated to it, as expected.

## 5 ALLOCATING CAPITAL TO POLICIES

Split  $k^l$  into  $n$  terms  $k^l = k_1^l + \dots + k_n^l$ , and write (4.1) as

$$\sum_{i=1}^n E \left[ \left( \frac{L_i}{1+r} - l_i - k_i^l \right) 1_\Omega \right] = d. \quad (5.1)$$

If a stand alone company writes only one policy with loss  $L_i$ , carries a capital amount  $k_i^l$ , and invests in the risk-free asset, then the value of default is  $L_i - (l_i + k_i^l)(1+r)$ , whenever  $L_i > (l_i + k_i^l)(1+r)$ . In our multi-policy company, consider  $L_i - (l_i + k_i^l)(1+r)$  as the value of default contributed by the  $i$ th policy, on the default set  $\Omega$ . So each term on the left-hand side of (5.1) is the (discounted) expected default value contributed by the  $i$ th policy. It should equal the economic default value  $d_i$  in (3.2),

$$E \left[ \left( \frac{L_i}{1+r} - l_i - k_i^l \right) 1_\Omega \right] = d_i. \quad (5.2)$$

Rewrite the equation as

$$l_i + k_i^l = \frac{1}{\text{Prob}(\Omega)} \left( E \left[ \frac{L_i}{1+r} 1_\Omega \right] - d_i \right), \quad (5.3)$$

or,

$$k_i^l = \frac{1}{\text{Prob}(\Omega)} \left( E \left[ \left( \frac{L_i}{1+r} - l_i \right) 1_{\Omega} \right] - d_i \right). \quad (5.4)$$

$k_i^l$  is the amount of capital allocated to the  $i$ th policy, and  $l_i + k_i^l$  the amount of asset allocated to the policy. Adding up (5.4) for all  $i$  we obtain (4.3).

We now compare (5.4) with other capital allocation methods in the actuarial literature. Typically an allocation process begins with selecting a risk measure. Adding a policy increases the total company risk, thus creates a higher capital requirement. Using the risk measure one calculates the additional capital, called the “marginal capital” of this policy. Then the total company capital is allocated in proportion to the marginal capitals. (See Venter [7].) Our approach does not directly measure the risk contributed by a policy. It focuses on the expected value of default. There is a unique, economically sound method to split up the expected default (3.2). Then the company capital can be correspondingly split up so that each policy contributes exactly the right amount of expected default. Derived directly from (3.2), the allocation (5.4) seems also economically sound.

An advantage of using the expected value of default as the basis for capital allocation is that closed-form formulas can be obtained. This first became evident in Myers and Read [4] and Butsic [1], although their solutions are flawed. (Incidentally, this author has proved that the “marginal change in capital” defined in [4] and [1] would equal  $k_i^l$  under their unrealistic condition  $d_i/l_i = d/l$ .) Sherris [6] also suggested two tentative criteria of allocating capital based on (3.2). One method assumes a common allocated-capital-to-liability ratio across all policies, the other uses the same ROE. These criteria seem too arbitrary. Our approach is more objective and the final result (5.4) more tractable.

Formulas (3.2) and (5.4) are stated for individual policies. But they also hold for any blocks of policies, such as lines of business or underwriting units. The allocation is additive: the expected value of default or the amount of capital allocated to a block of policies is the sum of that allocated to the individual policies.

Finally let us look at a special case. Assume loss  $L_i$  is certain in amount.

Then  $l_i = L_i/(1+r)$ . This policy is still subject to unrecoverable loss in the event of default. So  $d_i$  is some positive number. (5.4) implies  $k_i^l = -d_i/\text{Prob}(\Omega)$ . This negative capital means that such a policy helps reduce the company capital need.

## 6 A NUMERICAL EXAMPLE

We illustrate the use of (5.4) with a step-by-step calculation. The hypothetical company consists of lines of business taken from a real insurance company. The probability distributions of  $L_i, L, A_j$  and  $A$  are needed. In practice, the distribution of individual losses and assets are estimated from the historical data. Then these distributions are combined using selected correlations to produce the company aggregate loss and asset distributions.

Consider a company with three lines. Use simulation to generate 10,000 samples for each line's future loss. The 10,000 sample points are ranked from low to high, as shown in Exhibit 1. The mean losses and the standard deviations are calculated. ( $L_1$  has a relatively high standard deviation. It may represent a catastrophe coverage.  $L_1$  also has a heavier tail.) For simplicity we assume the company asset is invested risk-free, and the interest rate is 0. So the asset is constant  $A = a = l_1 + l_2 + l_3 + k$ ,  $k^a = 0$  and  $k^l = k$ . We also ignore the market risk of loss, so  $l_i = E(L_i)$ . Equations (3.1) and (3.2) reduce to

$$d = E[(L - a)1_{\Omega}], \quad (6.1)$$

$$d_i = E \left[ \left( L_i - \frac{L}{3}a \right) 1_{\Omega} \right]. \quad (6.2)$$

We introduce the correlation between the lines using the software package @RISK for Excel. The package uses a copula approach and can generate any specified rank correlation. (See Wang [10] for a comprehensive survey on correlation, including an introduction to copulas.) In @RISK, define three random variables  $X_1, X_2$  and  $X_3$ , each is uniformly distributed on the interval  $[0, 10000]$ . Judgementally choose the following (rank) correlations between the three lines:  $\rho(1, 2) = 0$ ,  $\rho(1, 3) = 0.1$  and  $\rho(2, 3) = 0.3$ . 10,000 samples of the correlated triplet  $(X_1, X_2, X_3)$  are simulated, as shown in Exhibit 2.

Exhibit 3 shows 10,000 sample points of a three-dimentional joint loss distribution. Its marginal distributions are  $L_1$ ,  $L_2$  and  $L_3$ , as in Exhibit 1, and its (rank) correlations are given by the  $\rho$ 's in Exhibit 2. The table is constructed using Exhibit 1 for sample points and Exhibit 2 as indices. For each row in Exhibit 2, say the first row, find the 4722nd  $L_1$ , 7706th  $L_2$  and 4754th  $L_3$  in Exhibit 1, and calculate their sum  $L$ . The four numbers constitutes one row in the table. The 10,000 rows are then sorted by  $L$ . Assume the capital amount  $k = 14,000$  (about 1/3 of the expected loss  $l = l_1 + l_2 + l_3 = 42,776$ ). An insolvency occurs when  $L > l + k = 56,776$ . In the chart, the insolvency is represented by the sample losses with rank greater than 9733 (below the line).

Exhibit 4 carries out the rest of the calculation using (6.1), (6.2) and (5.4). In the end the allocated capitals,  $k_i^l$ , are obtained. As expected, Line 1 has the highest allocated capital to liability ratio.

## 7 CONCLUDING REMARKS

This paper develops a method to allocating capital for the ratemaking purpose. In the premium formula (2.2),  $d_i$  and  $k_i$  are consistently calculated. (In later sections  $k_i = k_i^l$ .) We argue that only part of the capital should be allocated to the liabilities, and the rest to the risky assets. This prevents penalizing the policyholders for the company's decision to take on more investment risk. The allocated capitals, given by (5.4) and (4.6), depend on the joint distribution of the liabilities and the assets on the default set  $\Omega$ . By definition, capital allocation is more related to the "tail dependence" of the losses and assets, than to the variance and covariance of the variables. So our results are more reasonable than those in Myers and Read [4] and in Butsic [1].

It is tempting to use the same allocated capital to calculate the ROE. But further research is needed. For simplicity the following discussion assumes the collected premium is adequate. In this case the denominator, i.e., the allocated capital, is  $k_i^l$ . The numerator is the sum of the underwriting gain and the investment income. The underwriting gain is calculated as premium minus loss minus expense. But how to calculate the investment income

may be an issue. The principle of investment is the allocated asset  $l_i + k_i^l$ . The rate of return may be chosen between the risk-free rate and the actual investment yield. To be consistent with the capital allocation, the risk-free rate seems a more appropriate choice. It does not penalize the policyholders (or the managers of business units) for possible investment loss. This ROE calculation suits the purpose of ranking policyholders or business units. It may be necessary to tailor the calculation to fit other projects.

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## Exhibit 1

### Simulated Sample Losses by Line

Each line is individually simulated and ranked

Rank	$L_1$	$L_2$	$L_3$
1	0	119	551
2	0	1,294	4,076
3	0	1,837	5,414
4	0	1,982	5,778
5	0	2,166	6,462
6	0	2,272	7,043
⋮	⋮	⋮	⋮
9995	21,812	8,757	34,509
9996	21,825	8,758	34,542
9997	21,885	8,758	34,621
9998	21,889	8,781	34,624
9999	21,926	8,784	34,664
10000	22,128	8,784	34,679
Mean ( $\bar{l}_i$ )	10,128	7,112	25,536
Std Dev	5,217	1,170	5,661

## Exhibit 2

Correlated Uniform Distributions in [0, 10000]

Simulated with @RISK

Rank correlations:  $\rho(1, 2) = 0$ ,  $\rho(1, 3) = 0.1$ ,  $\rho(2, 3) = 0.3$

$X_1$        $X_2$        $X_3$

4,722    7,706    4,754

2,126    4,853    2,807

6,331    125    5,946

8,530    6,868    6,018

6,000    6,987    4,163

2,848    6,425    1,646

⋮      ⋮      ⋮

240    1,887    3,294

8,910    5,431    5,838

6,409    6,773    9,362

9,483    8,351    8,881

8,887    9,077    3,845

7,675    4,818    730

### Exhibit 3

Correlated Sample Losses

Marginals as defined in Exhibit 1

Correlations as defined in Exhibit 2

Ranked by total loss  $L (= L_1 + L_2 + L_3)$

Rank	$L_1$	$L_2$	$L_3$	$L$
1	3,149	3,836	551	7,536
2	2,601	3,986	4,076	10,664
3	0	6,510	7,729	14,239
4	2,754	1,982	9,732	14,468
:	:	:	:	:
9731	16,622	7,941	32,166	56,729
9732	17,502	7,382	31,874	56,758
9733	17,363	6,876	32,522	56,761
9734	17,353	7,847	31,589	56,790
9735	17,015	7,512	32,264	56,790
:	:	:	:	:
9997	20,812	7,191	33,923	61,926
9998	20,040	7,999	33,991	62,030
9999	20,476	7,683	34,051	62,211
10000	20,151	8,467	34,004	62,622
Mean Loss	10,128	7,112	25,536	42,776
Capital ( $k$ )				14,000
Asset ( $a$ )				56,776

Note:  $\Omega$  is represented by the samples with rank  $> 9733$ .

## Exhibit 4

### Calculation of Allocated Capitals

(1)	$l$	42,776
(2)	$k$	14,000
(3)	$l + k$	56,776
(4)	$\text{Prob}(\Omega)$	0.0267
(5)	$d$	42.39

		Line 1	Line 2	Line 3	Total
(6)	$l_i$	10,128	7,112	25,536	42,776
(7)	$d_i$	13.47	5.63	23.30	42.39
(8)	$E((L_i - l_i)1_{\Omega})$	217.72	19.24	179.23	416.19
(9)	$k_i^l$	7,649.97	509.74	5,840.29	14,000.00
(10)	$k_i^l/l_i$	0.76	0.07	0.23	0.33

Notes:

- (4) from Exhibit 3: number of samples below the line = 267.
- (5) from Exhibit 3: summing up all  $L$ 's below the line, less  $56,776 \times 267$ , divided by 10,000.
- (7) from Exhibit 3: summing up all  $L_i$ 's below the line; summing up all  $L_i/L$ 's below the line, times 56,776, subtracted out from the first sum, then divided by 10,000.
- (8) from Exhibit 3: summing up all  $L_i$ 's below the line, less  $l_i \times 267$ , divided by 10,000.
- (9) = ((8) - (7))/(4).