Leverage and the Cost of Capital in the Insurative Model

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Abstract

This paper addresses the challenge faced by practitioners in corporate finance and risk management in the current environment of increased scrutiny on corporate governance, risk assessment and capital adequacy. In particular, Chief Risk Officers face significant challenges in articulating Enterprise Risk Management (ERM) issues in a framework familiar to other financial managers. Using the Insurative Model first introduced in Shimpi (1999), we see that management has two additional forms of leverage – risk leverage and Insurative leverage – that are just as important as financial leverage in determining the capital structure of the firm. This leads to a more complete perspective of a firm's cost of capital as a function of its ERM decisions.

Keywords*: Insurative Model, leverage, cost of capital, enterprise risk management*

1. Introduction

Risk management is not a new concept, nor is it a new activity. Nevertheless, under the heading of Enterprise Risk Management (ERM), it has taken on greater importance in the current environment of increased scrutiny on corporate governance, risk assessment and capital adequacy. Corporate managers – including the new function of Chief Risk Officer – have been developing processes and procedures to comply with various governance rules that have been introduced in the last few years. However, they continue to face significant challenges in articulating ERM issues in a framework that would be familiar to most financial managers. The conventional statistical and actuarial risk analyses are not accessible to Treasurers, CFOs and CEOs.

There have been several efforts to bridge this know-how gap. Mayers and Smith (1982a, 1982b, 1987) describe why it makes sense for corporations to manage their risks through insurance. Froot, Scharfstein and Stein (1993, 1998) and Merton and Perold (1993) consider the influence of risk management decisions on the capital structure of financial institutions. The Insurative Model first described in Shimpi (1999) continues in that tradition, addressing the rationale for ERM through the relationship between risk and capital management for corporations in general. As we see in this paper, the model presents a positive theory for ERM that can be described as *providing shareholders with the opportunity for leveraged returns*.

Consider the following definitions:

- *Capital Management* delivers the optimal capital resources that are sufficient to support the needs of a firm and, in particular, to cover the risk exposures that the firm faces.
- *Risk Management* ensures that the firm's operational and financial exposures are controlled or structured in such a way that they are supportable by its capital resources.

The Insurative Model gives us an analytical framework that enables us to see the relationship between risk and capital management more clearly and make the assertion that they are identically the same problem. It departs from the conventional definition of capital resources, which refers to paid-up capital such as equity and debt, and takes the simply stated yet powerful position: *Insurance is a form of capital; it is contingent capital.*

To date, finance theory has treated insurance[1] as a *use* of capital since the insurance premium is an expense of the firm. The Insurative Model defines capital more broadly to include all financial instruments that substitute for equity. Capital can be either paid-up or contingent. The insurance premium is therefore a capital rental expense, in the same spirit as the interest on debt. This position provides an opportunity to tighten the relationship between risk and capital management.

Section 2 presents a statement of the problem. Sections 3 and 4 develop the Insurative Model and show how it addresses the problems described in Section 2. We see that that management has two additional forms of leverage – risk leverage and Insurative leverage – that are just as important as financial leverage in determining the capital structure of the firm. This leads to a more complete Insurative perspective of a firm's cost of capital in Section 5.

2. Statement of the problem

Define a *firm* as a collection of risky, productive activities. At any instant in time, t, this firm has the following attributes:

F1. It is owned by its *equity shareholders, E*

F2. It has a stated business plan which is executed by *managers*, acting as *agents* for the shareholders

F3. The productive activities under the business plan are represented by a set of *real assets*, A^R, that generate a *return on assets*, π^R. Some of these assets are *operating assets*, A^O, generating a *return on operating assets*, π^O, and the rest are *liquid assets*, A^L, generating a *return on liquid assets*, π^L.

F4. It is exposed to N *risk types*:, $\rho(n)$, $n = 1 \ldots N$

F5. Each risk type is partitioned into L *exposure layers*, $\varepsilon(P)$, $P = 1 \ldots L$

F6. These exposures collectively describe (although not necessarily quantify) the *firm risk*, R, an N x L matrix, where:

$$R = \{R(\rho, \varepsilon)\} \qquad (1)$$

F7. The firm manages its risks by either retaining them or transferring them (generally through insurance and hedging). Therefore the firm risk is the sum of the firm's *retained risk*, R^F and *transferred risk*, R^T:

$$R = R^F + R^T \qquad (2)$$

F8. The minimum *capital required* by the firm to cover its risks and meet its business objectives is Q

F9. Q is a function of R through the operator, $f\{\text{risk}\}$, so that:[2]

$$\begin{aligned} Q &= f\{R\} \\ &= f\{R(\rho,\varepsilon)\} \\ &= f\{R^F(\rho,\varepsilon) + R^T(\rho,\varepsilon)\} \qquad (3) \\ &= f\{R^F(\rho,\varepsilon)\} + f\{R^T(\rho,\varepsilon)\} \end{aligned}$$

assuming, for now, that the function f is additive

F10. The firm can choose between *M capital resources*, including equity, *E*. These capital resources are defined as *Insuratives.*

F11. The firm raises total *firm capital* of K to run its business, composed of a linear combination of the M Insuratives, each with value $K(m)$ and a corresponding cost of capital, $k(m)$, $m = 1 \ldots M$. Hence the *total average cost of capital* (*TACC* or τ) of the firm is the weighted cost given by:

$$\tau = \frac{1}{K}\sum_{1}^{M} k(m) \times K(m)$$

where (4)

$$K = \sum_{1}^{M} K(m)$$

F12. The firm operates on a marked-to-market basis, i.e. all book values are also market values. At the date that capital is raised or a liability incurred, book value equals market value by definition. At later dates, the market value of assets and liabilities incurred in a previous period can differ from the accounting book value, particularly when various accounting conventions are applied. Furthermore, market values are not always reliably calculated for all classes of assets and liabilities. For the purposes of this paper, these limitations are ignored, although in practice they need to be considered and handled appropriately.

The managers want to have an appropriate capital structure given the activities of the firm and the risks it faces. At the same time, they must ensure that they are managing the firm's risks appropriately. Given these objectives, the central problem for this firm can be stated as follows:

Problem 1: Capital management

P1. What is the optimal capital structure of the firm, expressed as a linear combination of a portfolio of the M available capital resources, which minimizes cost, τ?

This question has been addressed extensively in the finance literature. Nevertheless, there are some fresh aspects to be considered. Several other related problems flow from this central problem statement and these are listed below:

Problem 2: Capital adequacy

P2. What is the minimum capital required by the firm, Q?

Problem 3: Capital resources

P3. What are the capital resources available to the firm, M?

Problem 4: Risk management

P4. What is the optimal insurance/hedging strategy which determines both the partition between retained risk, R^F and transferred risk, R^T, and the financial instruments to realize that partition?

Problem 5: Analytical framework

P5. What analytical framework enables decision-making under uncertainty with sparse data in order to address comprehensively both risk and capital management problems?

This paper presents the Insurative Model as the analytical framework through which problems P1 to P4 are addressed. Problem P5 is beyond the scope of this paper and will be addressed separately.

3. The role of corporate capital

Attributes F4, F5, and F6 describe the risks of the firm and F8 and F9 describe the capital required by the firm to finance its activities and respond to risks. In other words, a firm

needs capital to fund its operations and to cushion it against adverse financial results. In addition, it may need an additional amount to assure observers of its financial soundness.

From this perspective of the *role* of capital, the *capital required* by a firm at any instant, Q, can be defined as:

$$
\begin{aligned}
&Q = Q^E + Q^S = Q^O + Q^R + Q^S \\
&\quad \text{where} \\
&Q^E = \text{Economic capital} \\
&Q^O = \text{Operational capital} \\
&Q^R = \text{Risk capital} \\
&Q^S = \text{Signaling capital}
\end{aligned}
\qquad (5)
$$

From the perspective of the *amount* of capital required, the preceding implies that it will be a function of the risk exposures of the firm. Let us define the function $f\{risk\}$ to indicate the amount of capital necessary to cover some given "risk".[3] From Equation 1, the firm is exposed to *firm risk, R*, denoting the risk of the all its activities. From Equation 3, we see that firm capital, which is the minimum amount of capital necessary to cover firm risk, Q, is determined as $f\{R\}$.

If we define $\overline{R}$ to represent the expected exposure to firm risks, then operational capital is:

$$Q^O = f\{\overline{R}\} \qquad (6)$$

In a static model of a firm, this is the basis on which the firm's required capital is calculated to cover the expected risk exposures *with certainty*. Note that the expected exposure, $\overline{R}$ can be defined to be something other than the mathematical expectation, for example the mathematical expectation plus some margin (say, 2 standard deviations). The point is that Q^O is calculated to guarantee coverage of the specified expected exposure. Trivially, if the firm undertakes only risk-free projects, Q^O will be the minimal capital it needs to stay in business.

In a dynamic economic capital model of a firm it is critical to consider not just the expected exposure to risk but also the variations in the exposure across all forms of risk, ρ. A firm will require risk capital, Q^R, in addition to its operational capital, Q^O, to cover the financial

consequences of the risks to its activities. The sum of these two items gives the economic capital of the firm Q^E. From Equation 3, we see that in the absence of signaling capital:

$$Q^E = f\{R\} \qquad (7)$$

from which:

$$Q^R = Q^E - Q^O = f\{R\} - f\{\overline{R}\} \qquad (8)$$

Unlike operational capital which, by definition, is calculated to cover the expected risk exposure with certainty, risk capital cannot assure with certainty that all the firm's risk exposures are covered. Risk capital can only cover the firm's risk exposures up to a certain point, which we denote as the firm's *risk tolerance*, θ. As the firm's risk tolerance increases, Q^E and Q^R decrease, and vice versa.

In order to recognize the dependence of economic capital on risk tolerance, Equation 7 should be written as:

$$Q^E = f\{R|\theta\} \qquad (9)$$

Although θ is integral to the Insurative Model, the actual form of θ is not fixed. There are a number of ways to articulate the risk tolerance. For instance it can be calculated as the capital needed to keep the firm's probability of ruin over some period below some defined level. For now we will take θ as given and continue to use Equation 7 which does not show explicitly the dependence of Q^E on θ.

Often it is not sufficient that only the managers of a firm be satisfied with the adequacy of the capital they have to cover their risks. Additional signaling capital is held to reassure outsiders that the firm is indeed as strong as the managers know it to be. In the examples to follow, we assume that there is no requirement to hold signaling capital, so that required capital is economic capital.

$$Q^S = 0 \Rightarrow Q = Q^E \qquad (10)$$

When considering P3 (capital resources), the available forms of capital (Insuratives) include both paid-up capital resources, M^P, and contingent capital resources, M^C. The firm can raise paid-up capital, K^P, from M^P and contingent capital, K^C, from M^C such that:

$$K = K^P + K^C \qquad (11)$$

The amount of capital actually raised (K) should be based on the amount of capital the firm requires (Q), in both paid-up and contingent forms (Q^P and Q^C). If we assume that the firm raises exactly the amount and type of capital that it needs then, as we will see in the next section, the Insurative Model states:

$$\begin{aligned} K^P &= Q^P = \alpha \times f\{R^F\} + \beta \times f\{R^T\} \\ K^C &= Q^C = (1-\alpha) \times f\{R^F\} + (1-\beta) \times f\{R^T\} \end{aligned} \qquad (12)$$

This means that a proportion α of retained risk and β of transferred risk is covered by paid-up capital and that the rest is covered by contingent capital. This equates all firm capital to the amount necessary to cover all firm risks, both retained and transferred. These equations tie P1, P2, P3 and P4 together. In effect, the Insurative Model provides a comprehensive view of corporate capital structure with a direct connection to risk management.

4. Building an integrated framework

The following sections develop the Insurative Model from first principles. The range of capital resources available to a firm is discussed and contingent capital instruments such as insurance are shown to qualify as capital. Consequently, firms have other forms of leverage besides the conventional financial leverage. The term Insurative is defined explicitly and forms the basis on which the Insurative Model is defined. In order to include contingent capital appropriately, the concept of Equity Value Equivalent (EVE) is described.

4.1 Capital resources

So far we have not discussed the financial instruments that constitute a firm's capital. The most basic form of capital is equity and most firms start out their existence financed only by equity. We use an all-equity firm, defined as follows, as a benchmark to consider alternative capital structures:

Baseline firm: A firm that has only equity capital, E^B, with value $K(E^B)$ equal to Q, sufficient to bear firm risk, R, within the risk tolerance, θ, of the firm:

$$K(E^B) = Q = f\{R|\theta\} \qquad (13)$$

The baseline firm has no debt, insurance or other capital resources. The cost of capital of the baseline firm is the cost of equity which, in equilibrium, is the return on equity. Traditionally, corporate finance is used to dealing with financial leverage where debt substitutes for equity. The Insurative Model proposes that a wider set of capital resources can be used to substitute for equity. From the firm's attributes F10 and F11, we see that the firm can choose between *M capital resources* (or *Insuratives*), to raise *firm capital* of *K*. in either paid-up or contingent form.

4.1.1 Contingent capital

It is easy to think that capital is limited to the various classes of equity and corporate debt where an investor pays cash immediately for a security interest in the firm, i.e. *paid-up capital*, M^P. This is the capital that appears on a firm's balance sheet. Most of corporate finance literature focuses on these forms of capital. A firm also has the ability to access capital in the future, i.e. *contingent capital*, M^C, in order to fulfill the same objectives as it does with paid-up, on-balance-sheet capital. Contingent capital is also referred to as off-balance-sheet capital since it does not appear on a firm's conventional balance sheet.

There are two broad forms of contingent capital. The distinction between them is based on whether some component of firm risk is retained or transferred as a part of the transaction:

1. <u>Committed Capital – Risk retained by firm</u>

   This form enables a firm to buy the right to access capital in case it is needed. Such a transaction can be a cost-effective way for a firm to postpone putting capital on its balance sheet until it needs to.

   One example is a bank line of credit. It does not form part of the firm's paid-up capital, since it is not yet paid to the firm. A fee is paid for the credit line, which is a premium for the option to raise capital in the future. If the firm utilizes the line and borrows from the bank, the loan at that time is treated as paid-up capital. Subject to some terms and conditions, the firm can draw cash from the facility whenever it desires. The bank takes the risk that the firm is able to repay the loan once it has drawn on the credit facility.

In contrast, the insurance market has seen the development of *triggered* committed capital facilities. The option to draw cash from the facility is available only when a pre-determined event has occurred.[4] The firm retains the risk of the triggering event. The capital provider takes first, the risk that the triggering event may occur and second, the post-event credit risk which determines the firm's ability to repay the cash drawn.

2. Insurance/Hedge – Risk transferred by firm

The second way to access contingent capital is to transfer risks to other firms, thereby altering the retained risk profile and the consequent capital structure of the firm. The firm does not have to keep any paid-up capital to cover the transferred risk, except for some operational capital to pay the on-going insurance premium. All losses arising from that risk are borne by the insurer. The same principles apply when hedging in the derivatives market.

The distinction between these two forms of contingent capital is fairly straightforward. With the first, the firm still owns the risk, and can raise paid-up capital at some later date if it needs to. With the second, the firm no longer owns the risk, and so does not need to raise any paid-up capital to cover it. Instead it gets a payment from the insurer to cover the financial consequences of the event. In both cases, the firm gets cash later, when it needs it.

4.1.2 Is insurance capital?

With paid-up capital, the "cash now" aspect qualifies it as capital. Since contingent capital is clearly "cash later" does that qualify as capital as well? The answer lies in understanding the role of capital.

A fundamental rationale for capital is to bear a firm's exposures to risk over the lifetime of its productive activities. There are exposures in the current period and in future periods. In a new baseline firm, the amount of equity raised in the current period must equal Q so that it is able to bear the firm risks. In the current period, it can use debt to substitute for equity. Under Modigliani-Miller (MM) assumptions, the total debt and equity capital must equal Q.

Instead of substituting equity with "cash now", the firm can opt to substitute some of its equity with "cash later" facilities. Using contingent capital, it pre-arranges to have cash

delivered in the future as the need arises. These pre-arrangements do not appear in the conventional financial statements, although there may be references in the notes to the accounts. They are economically valuable since they clearly enable the firm to forego raising equity in the current period.

It is understandable that CFO's try to minimize the insurance expense since they cannot show the value of the protection on their financial statements. There is a major challenge in recognizing insurance as capital – that of valuation. Data and analytical tools are just not available to develop credible values uniformly. However, this has always been a limitation at the early stages of any new development. If mandated, data can be collected and valuations, even crude ones, can be made. After all, the aim of the increased disclosure on derivatives and stock options is to do just that.

The value of contingent capital must be discounted for the risk that the cash does not appear when expected due to the non-performance of the counterparty (e.g. the insurer is unable to pay a claim when it is bankrupt). In effect, entering into such a transaction changes the underlying risk exposures of the firm, since each contract both increases and decreases risk to the firm. This is not unique to contingent capital. Paid-up capital (e.g. debt) has a similar effect, specifically the increase in likelihood of bankruptcy due to the addition of a fixed charge (interest on debt) that the firm must pay.

The rationale to treat insurance and other forms of contingent capital as capital is therefore fairly straightforward. Consider a firm that is able to use equity, debt and insurance to cover its risk exposures within its risk tolerance:

1. Like equity and debt, insurance covers some portion of a firm's risks.
2. Using insurance allows a firm to reduce the amount of equity it needs; if an insured firm cancels its insurance it would have to raise more equity to maintain its ability to bear the same risks.
3. The premium paid for insurance is the cost of renting contingent capital, just as debt interest is the cost of renting paid-up capital.

4.1.3 Firm capital and firm risk

The capital resources available to the firm include both paid-up capital, M^P and contingent capital, M^C. The amount of capital available to the firm is correspondingly K^P and K^C so that firm capital, as stated in Equation 11, is $K = K^P + K^C$

The amount of capital the firm should raise depends on the amount of capital it requires, Q, which in turn depends on the risks it faces, R. The preceding discussion now allows us to relate the capital resources that a firm has put in place, K, to its required capital, Q, which is a function of its risks, R as follows.[5]

Capital Adequacy: If at any time, t, when K and Q are calculated, taking into account possible future scenarios over time, the firm's capitalization at time t is determined as follows:

$$\begin{aligned} &K = Q \Rightarrow \text{firm is adequately capitalized} \\ &K > Q \Rightarrow \text{firm is overcapitalized} \\ &K < Q \Rightarrow \text{firm is undercapitalized} \end{aligned} \qquad (14)$$

An overcapitalized firm is likely not meeting investor expectations since it has more capital than is economically necessary. An undercapitalized firm is taking risks beyond its stated tolerance and, at the extreme, can be exposing itself to insolvency.

The firm also makes a decision on the risks it keeps and the risks it transfers, which we formally define as follows:

Transferred risk, R^T: Subset of firm risk, R, for which losses are borne by another party. The firm pays only the cost of transferring this risk. If a loss event occurs, the other party pays the firm the amount of the loss.[6]

Retained risk, R^F: Subset of firm risk, R, for which losses are borne by the firm.

These definitions reflect the focus on potential losses due to the risk exposures. Clearly these risk exposures also create the opportunity for gain. Since risk tolerance reflects the firm's ability to withstand losses, the focus of these definitions is appropriate.

Equation 5 defined Q in terms of the role of capital. From Equations.2 and 3, we can also define Q in terms of the firm's risk management decision as follows:

$$
\begin{aligned}
&Q = Q^F + Q^T \\
&\text{where} \\
&Q^F = f\{R^F\} = \text{capital required for retained risk} \\
&Q^T = f\{R^T\} = \text{capital required for transferred risk}
\end{aligned}
\qquad (15)
$$

As the firm determines its capital structure decisions, it needs to ensure that it has the right capital of the right type as well. It follows from Equation 14 that the firm should raise sufficient capital to cover its retained risks, K^F and to cover its transferred risks, K^T so that:

$$
\begin{aligned}
&K^F = Q^F \\
&K^T = Q^T \\
&\text{and} \\
&K = K^F + K^T
\end{aligned}
\qquad (16)
$$

4.1.4 Forms of Leverage

The equity holders (E) can earn a greater return if they lever the firm and use other capital resources. Any review of finance literature would seem to indicate that the capital structure discussions have considered leverage fairly completely. That is perhaps true of financial leverage (where debt, for instance, substitutes for equity). However there are additional concepts of leverage that are relevant in a firm's decisions on risk and capital.

We begin by defining financial leverage:

Financial leverage, φ: The portion of paid-up capital, K^P, that is not equity, $K(E)$:

$$
\begin{aligned}
\varphi &= \frac{K^P - K(E)}{K^P} \\
&= \frac{K(D)}{K^P}
\end{aligned}
\qquad (17)
$$

In a firm where paid-up capital is made up of only equity and debt, the numerator is the amount of debt, $K(D)$.

There are two additional forms of leverage. The first is Insurative leverage which is the proportion of capital that is contingent K^C relative to the total firm capital K. The second is risk leverage which describes the degree of risk transfer utilized by the firm.

Insurative leverage, η: The portion of paid up capital, K, that is contingent capital, K^C:

$$\eta = \frac{K^C}{K} \qquad (18)$$

The baseline firm has only paid-up capital, so $\eta = 0$. A firm that uses only financial leverage, i.e. only paid-up capital such as debt, has no contingent capital, so again $\eta = 0$. A firm that uses both financial and Insurative leverage will have $\eta > 0$.

Risk leverage, λ: The portion of firm risk transferred by the firm, measured by the ratio:

$$\lambda = \frac{K^T}{K} \qquad (19)$$

The baseline firm always retains all the risk, so $\lambda = 0$. A firm that uses only financial leverage, i.e. only paid-up capital could be using some of that capital to transfer risk (e.g. a catastrophe bond or a debenture tied to a particular project), so $\lambda \geq 0$. A firm that uses only Insurative leverage need not be transferring risk (e.g. if it only uses committed capital facilities), so again $\lambda \geq 0$.

For most firms, $\eta \neq \lambda$. This is demonstrated by the observation that most firms have a bank line of credit (part of K^C but not K^T) and an insurance policy (part of K^T but not K^C).

4.2 Defining Insurative

The preceding discussion shows that there are two basic strategies for raising capital (cash now or cash later) and two basic strategies for managing risk (retaining it or transferring it). The conventional definitions of capital and risk do not explicitly relate to each other and therefore do not recognize these basic strategies. The firm attributes F4, F5 and F6 define risk types, ρ, and exposure layers, ε, that collectively describe firm risk $R(\rho, \varepsilon)$. Let us define capital and risk as follows:

Capital: Resource available to a firm to finance its corporate activities

Risk: Exposure that impairs a firm's ability to achieve its corporate objectives

We combine these elements to formally define the M capital resources available to a firm as Insuratives in a way that explicitly links capital and risk as follows:

Insurative: Capital resource, $m(\rho,\varepsilon,\delta)$ available to finance a firm's risk exposures, where $m = 1$ to M.

The 3 parameters that describe an Insurative, m, are:

1. ρ = Risk-type that it is covering,
2. ε = Exposure to the risk-type covered, and
3. δ = Dedication of capital to the firm

The firm's attribute F4 states that the firm is exposed to N risk types, $\rho(n)$, where $n = 1$ to N. The risk type effectively partitions the risks of the firm. Minimally the firm can be partitioned into two risk types – those that are retained, R^F, and those that are transferred, R^T. In practice, the partitions are likely to be more extensive and granular, e.g. oil price risk and California earthquake risk. An Insurative can cover one or more risk types.

The firm's attribute F5 states that each risk type can be partitioned into L exposure layers, $\varepsilon(P)$, where P=1 to L. The risk exposure is another way to partition risks. Partitioning risk in this manner is standard practice in both insurance and derivatives markets. In insurance, for instance, a risk exposure will be partitioned into first loss layer, second loss layer and so on. In derivatives markets, a risk exposure can be partitioned into in-the-money and out-of-the-money. This is not as passive a partitioning as it appears. The exposure definition can be as broad as necessary. A time dimension is easily incorporated and so is location.

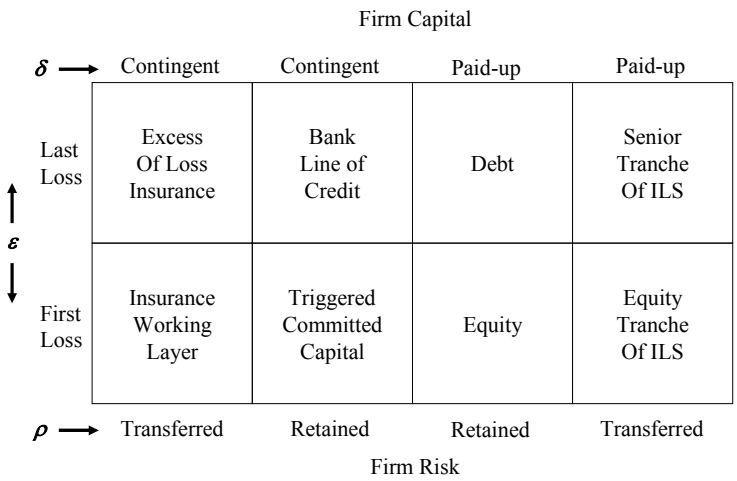
The dedication parameter, δ, specifies whether the capital is paid-up and immediately available (100% dedicated to the firm) or contingent and potentially available to the firm (partially or not at all dedicated to that firm).

Assume that each parameter has only 2 values. Then there are 8 combinations of parameters. ($2^3 = 8$). Table 1 and Figure 1 present examples of these 8 Insuratives, most of which are familiar. Note that at this broad level, there are several candidates for many of the categories; only one was picked for illustration. An Insurative does *not* define a new type of security. Instead, it is a new way to think about capital resources that are already familiar and perhaps help to develop new ones. It is a way to capture the richness of the capital resources available to a firm and to understand the role each Insurative plays in first, being part of the capital structure and second, covering the risk structure of the firm.

Table 1 Examples of Insuratives

Insurative, m	Risk-type, ρ	Exposure, ε	Dedication, δ
Equity	Retained	First loss	Paid-up
Debt	Retained	Last loss	Paid-up
Committed capital	Retained	First loss	Contingent
Bank credit line	Retained	Last loss	Contingent
Equity-tranche of ILS[7]	Transferred	First loss	Paid-up
Senior tranche of ILS	Transferred	Last loss	Paid-up
Insurance working layer	Transferred	First loss	Contingent
Excess of loss cover	Transferred	Last loss	Contingent

Figure 1 Examples of Insuratives



The capital that a firm has is sum of the capital provided by each of its Insuratives. If we define $K(m)$ to be the capital provided by an Insurative, then the firm capital is:

$$K = \sum_{m=1}^{M} K\big(m(\rho,\varepsilon,\delta)\big) \qquad (20)$$

When $\delta = 1$, capital is dedicated to the firm and is paid-up. When $\delta < 1$, capital is committed to the firm, but not dedicated solely to it. We can therefore define K^P and K^C as follows:

$$K^P = \sum_{m=1}^{M} K\big(m(\rho,\varepsilon,1)\big) \qquad (21)$$

and

$$K^C = \sum_{m=1}^{M} K\big(m(\rho,\varepsilon,\delta)\big) \quad \text{for } \delta < 1 \qquad (22)$$

4.3 Models of capital structure

Figure 1 illustrates the 3-dimensional view of capital structure delivered by the Insurative Model. First is the *type of dedication of capital*, paid-up or contingent, second is *type of risk*, retained or transferred, and third is the *type of exposure*, first-loss or last-loss.

The preceding sections have assembled several of the components necessary to state the Insurative Model. There are two approaches that can then be taken to compare that to two other models. The first is to show that the other models are special cases of the Insurative Model. The second is to develop the Insurative Model as a logical synthesis of the other models. This section takes the latter approach.

Traditionally, there are two models of finance – the Standard Model of corporate finance and the Insurance Model. By combining these two, the Insurative Model is demonstrated to be a richer framework than the mere addition of the two.

4.3.1 The Standard Model

The *Standard Model* is the conventional corporate finance approach to developing a firm's capital structure. The firm's risk is not explicitly specified. Instead, the starting point is a statement of the firm's paid-up capital requirement. The objective is to construct a combination of equity, mezzanine (or subordinate) debt and senior debt that an investment or commercial bank can sell to investors or syndicate to lenders. Normally, the distinction between these forms of capital is understood in terms of priority to claims on corporate cash flows while the firm is operating, and on corporate assets if the firm is liquidated.

Effectively, K^C is not contemplated. K is defined to be K^P. There is no explicit relation to firm risk, so R, θ and Q are not addressed. In terms of the problem statements, P1 (capital management) and partially P3 (capital resources – but only paid-up) are considered.

There is another way to describe this model, one that relates to risk. This model only refers to risks that are retained by the firm. It addresses only on-balance-sheet, paid-up capital. If the firm uses only equity capital, then the shareholders are fully exposed to the firm's retained risks. By introducing senior and subordinate debt, the model distinguishes between the forms of capital by their exposure to the firm's risks. The Standard Model stratifies all

the firm's risks and specifies that capital providers share those risks sequentially. The senior debt providers are the least exposed, and the equity investors are the most exposed.

In effect, the Standard Model equates the paid-up capital of the firm to the amount necessary to cover its retained risk. It ignores the possibility that other forms of capital may be utilized to cover such risk.[8] For the Standard Model to be a true representation of the firm, the following must hold:

$$\begin{aligned} &R^T = 0 \\ &K^C = 0 \\ &R = R^F \\ &Q = Q^F = f\{R^F\} \\ &K = K^P \\ &K^P = Q^F \end{aligned} \qquad (23)$$

Assuming that the firm has only equity, E, and debt, D, then the general cost of capital Equation 4 takes the specific form of the WACC equation:

$$w = k(E) \times \frac{K(E)}{K(E)+K(D)} + k(D) \times \frac{K(D)}{K(E)+K(D)} \qquad (24)$$

Under MM assumptions, since w is constant for a given risk class (in this case for a stated θ), then $k(E)$ and $k(D)$ must vary as financial leverage, φ, is changed.

4.3.2 The Insurance Model

It is not common to consider insurance in a corporate finance setting. Nevertheless, doing so provides some insights. In the Insurance Model, risks are specified explicitly. Risks are either transferred through insurance or hedging or retained by the firm. Transferring risks changes the firm's retained risk profile. As a consequence insurance has a direct impact on corporate capital structure. Insurance is contingent, off-balance-sheet capital that covers the risks transferred by the firm

In effect, the Insurance Model equates the contingent capital of the firm to the amount necessary to cover its transferred risk. It focuses only on the risk being transferred, R^T, and the cost of doing so (i.e. the *insurance premium*, p). In terms of the problem statements, P4

(risk management) and partially P3 (capital resources – but only contingent) are considered. The following describes the Insurance Model:

$$\begin{aligned} &R^F > 0, \text{ but ignored} \\ &K^P > 0, \text{ but ignored} \\ &Q^T = f\{R^T\} \\ &K^C = Q^T \end{aligned} \qquad (25)$$

The Insurance Model does not address the amount of capital represented by insurance, K^C. Consequently there is no corresponding concept of cost of insurance as a function of K^C that can be used in the cost of capital Equation 4. The insurance premium is <u>not</u> the cost of insurance; it is the price or expense amount. (This is similar to debt interest, $k(E)$ H $K(E)$ which is an expense amount, and which is not the same as the cost of debt, $k(E)$, a rate as a percentage of debt capital.) To overcome this limitation, we need to develop a corporate finance view of the Insurance Model and a new measure, the Equity Value Equivalent (EVE) of capital. This is done by the Insurative Model.

4.3.3 The Insurative Model

In this section we define the Insurative Model and show how EVE delivers the value, K^C.

4.3.3.1 Definition

Combining the effects of the Standard Model and the Insurance Model gives a generalized framework, the Insurative Model, to consider the effects of on- and off-balance-sheet capital, accessing both the insurance and capital markets.

As a first step, combining Equations 23 and 25 enables us to equate all firm capital to the amount necessary to cover all firm risks, both retained and transferred.

$$\begin{aligned} &\text{Standard Model:} && \text{Insurance Model} \\ &Q^F = f\{R^F\} && Q^T = f\{R^T\} \\ &K^P = Q^F && K^C = Q^T \\ &\qquad \text{Sum of the 2 models:} \\ &\qquad Q = Q^F + Q^T = f\{R^F\} + f\{R^T\} \\ &\qquad K = K^P + K^C = Q^F + Q^T = Q \end{aligned} \qquad (26)$$

Although this is useful, the Insurative Model is structurally richer than the addition of the two models. In the Standard Model, paid-up capital only referred to retained risk and in the Insurance Model, contingent capital only referred to transferred risk. In the Insurative Model, paid-up capital can be used to cover some of both retained and transferred risks. Likewise, contingent capital can be used to cover some of both those risks as well. To see this, let paid-up capital cover proportion α of retained risk and β of transferred risk, so that contingent capital covers the rest. Then the following relationships define the Insurative Model and hold for an adequately capitalized firm:

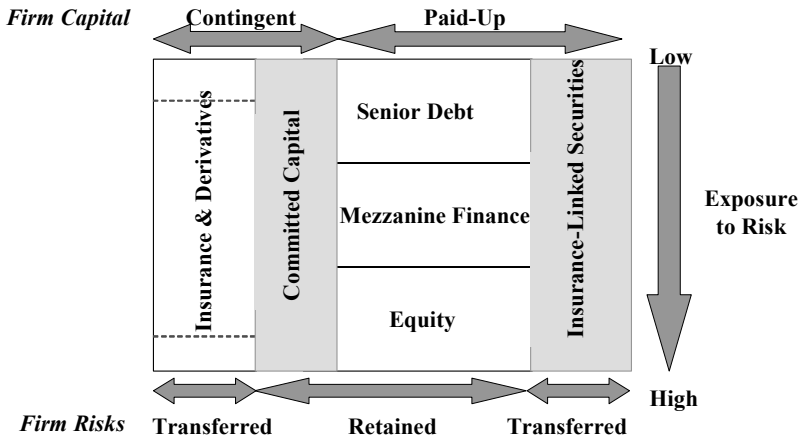
$$
\begin{aligned}
&\underline{\text{Insurative Model}}: \\
&K^P = \alpha \times f\{R^F\} + \beta \times f\{R^T\} \\
&\quad = \alpha Q^F + \beta Q^T \\
&\quad = Q^P \\
&K^C = (1-\alpha) \times f\{R^F\} + (1-\beta) \times f\{R^T\} \qquad (27) \\
&\quad = (1-\alpha) Q^F + (1-\beta) Q^T \\
&\quad = Q^C \\
&\text{Summing gives}: \\
&K = K^P + K^C = Q^F + Q^T = Q^P + Q^C = Q
\end{aligned}
$$

These equations address 4 of the 5 problem statements, P1, P2, P3 and P4. The Insurative Model framework captures the economics of conventional insurance and corporate finance instruments as well as the newer, integrated, alternative risk transfer products.

What implications does this model have for our picture of corporate capital structure? Figure 2 shows just how rich the diversity of corporate capital resources can be, and how necessary it is to develop this generalized model of corporate capital and risk. The reality is more interesting than the figure implies since the lines between the boxes are not always so clear; a single transaction may have components that cut across several of the boxes.

The term *Insurative* was coined to refer to *any* corporate capital resource, be it debt, equity, insurance, derivative, committed capital or other. The Insurative Model embraces all of these instruments and allows us to evaluate their effectiveness in a consistent framework. Each instrument carries some defined set of risks of the firm and it is in that sense it is insurance or a derivative that hedges those risks – hence the word *Insurative.*

Figure 2 The Insurative Model



4.3.3.2 Equity Value Equivalent (EVE)

One of the challenges with insurance, derivatives and other such contingent capital instruments is ascribing a value to them. This is not an issue in the Standard Model, which simply ignores them, nor in the Insurance Model, which only focuses on the price of the instruments. This concept of the Equity Value Equivalent (EVE) is used to develop the value of these instruments.

Equity Value Equivalent (EVE) of an Insurative is the amount by which the equity of a firm can be reduced by the addition of that Insurative, while maintaining the firm's capital adequacy.

EVE of Insurative m:

$$K(m) = K(E^{old}) - K(E^{new})$$

where (28)

$K(E^{old}) =$ Value of equity without m

$K(E^{new}) =$ Value of equity with m

When adding a single insurance policy, for example, to the baseline firm, E^{old} will be E^{B} and E^{new} will be the reduced equity given the addition of m. When adding an Insurative to a firm with existing insurance coverage, then E^{old} will be the equity given the existing insurance and E^{new} will be the reduced equity given the existing insurance plus the new Insurative.

Since the capital structure of a firm comprises a portfolio of Insuratives, the order in which the Insuratives are added to the portfolio will have a material impact on the calculation of EVE for each of them. This is the same challenge that is encountered in other settings where individual values need to be ascribed to members of a portfolio, e.g. in allocating overhead costs to several product lines in a manufacturing firm or allocating capital to different lines of business. So long as the portfolio value is not distorted, there can be a variety of ways in which value is ascribed to any individual component. (Examples of EVE methods are beyond the scope of this paper.)

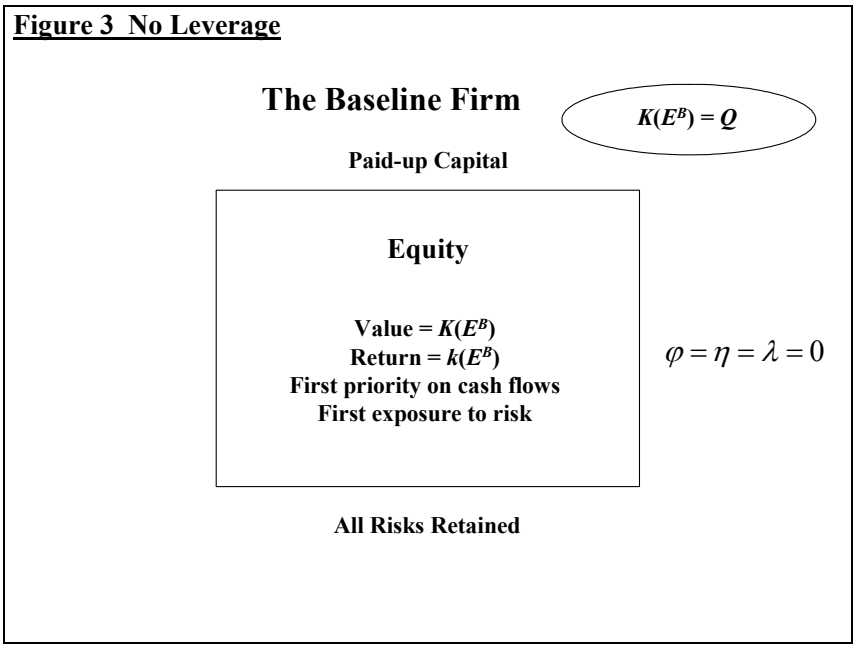
5 Insurative perspective on cost of capital

Equation 4 introduced TACC. In this section we take a closer look at TACC and compare it to WACC used in the Standard Model. In developing the Insurative Model, we stated that each Insurative carries a risk of the firm and therefore functions as insurance or a derivative. The concept of an insurance/derivatives perspective of securities issued by a firm is not new, although the approach taken below is different.

The idea that a firm's stock is an option on the underlying assets was suggested in the original Black-Scholes (1973) article, but it was actually developed into the notion that is familiar in finance today by Merton (1974). In this view, the claims of shareholders and bondholders are distinguished by default risk as a put option that is written (in effect, if not explicitly) by the bondholders and held by the shareholders. In other words, shareholders own a put option, reflecting their limited liability. If asset values fall below the face value of bonds, the shareholders simply turn over (put) the assets to the bondholders and walk away.

This characterization of the bondholders as the put writers captures the termination criteria, i.e., effectively the insurers in bankruptcy, and is analytically accurate but not necessarily intuitively appealing. First, in the event of default, the bondholders get the firm and the underlying assets while the shareholders get nothing. That sounds like the bondholders are the ones who are protected by the shareholders. After all, when you buy insurance, the insurance company shouldn't get your firm when a claim occurs, leaving you with nothing.[9] Secondly, it is not clear what insurance premium the shareholders paid to the bondholders.

The Insurative Model's treatment of risk and capital in the same framework creates a more appealing description of the relationship between the various capital providers over time, not just at termination. In the process it shows that the WACC and TACC are natural consequences of this relationship.



First consider the Standard Model setting with the baseline firm that has only equity (Figure 3). The firm has underlying productive assets A. Assuming no debt or insurance the minimum equity (baseline capital) required for this firm is Q, which will earn a rate of q. With no excess capital, Q must equal A. Shareholders contribute equity of E^B, with value of $K(E^B)$, which must equal Q for capital adequacy. (In the following, superscripts S and I are used to denote Standard Model and Insurative Model values for equity, debt and insurance.)

The equity holders decide to lever the firm with debt, D^S (Figure 4). This reduces equity from E^B to E^S. The value of debt is $K(D^S)$ and that of equity is $K(E^S)$, so $K(D^S) + K(E^S) = Q$.

The firm's risks are specified as a vertical tower that is stratified horizontally, i.e. the risks of the firm are all retained by the investors, but shared with different exposures. Equity holders demand a return of $k(E^S)$ and debt holders demand $k(D^S)$. Since they are both investors in the firm, one can argue that they should both get the same return on the assets of the firm, q. However, since the equity holders take the first loss position, they are protecting the debt

holders. The equity holders are entitled to an extra return for the risk that they are taking. Similarly, the debt holders give up some of their return of q. This is summarized as:

$$\begin{array}{lccccc} \underline{\text{Risk taker}}: & \underline{\text{As investor}} & & \underline{\text{Extra}} & & \underline{\text{Return}} \\ \text{Equity}: E^S \Rightarrow & q & + & \{k(E^S)-q\} & = & k(E^S) \\ \text{Debt}: D^S \Rightarrow & q & - & \{q-k(D^S)\} & = & k(D^S) \end{array} \qquad (29)$$

In this setting, $k(D^S) < q < k(E^S)$. The extra $\{k(E^S) - q\}$ for the equity holder looks like an insurance premium that is received for taking on more risk. The reduction of $\{q - k(D^S)\}$ for the bond holders looks like an insurance premium that they pay so that they can get priority in distribution of assets before the shareholders. For this to be internally consistent at equilibrium, the two amounts must equate.

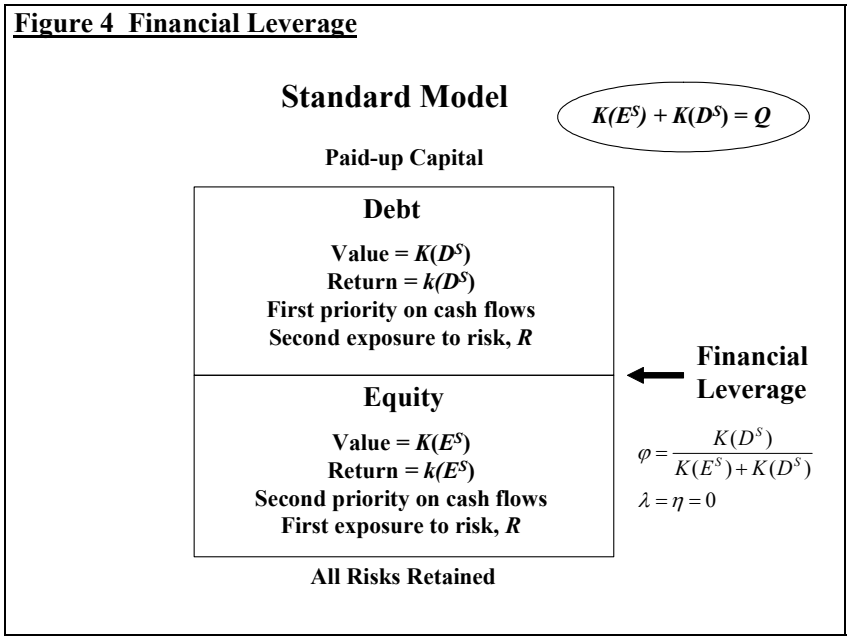
$$K(E^S)\times\{k(E^S)-q\} = K(D^S)\times\{q-k(D^S)\}$$

from which (30)

$$q = k(E^S)\times\frac{K(E^S)}{K(E^S)+K(D^S)} + k(D^S)\times\frac{K(D^S)}{K(E^S)+K(D^S)}$$

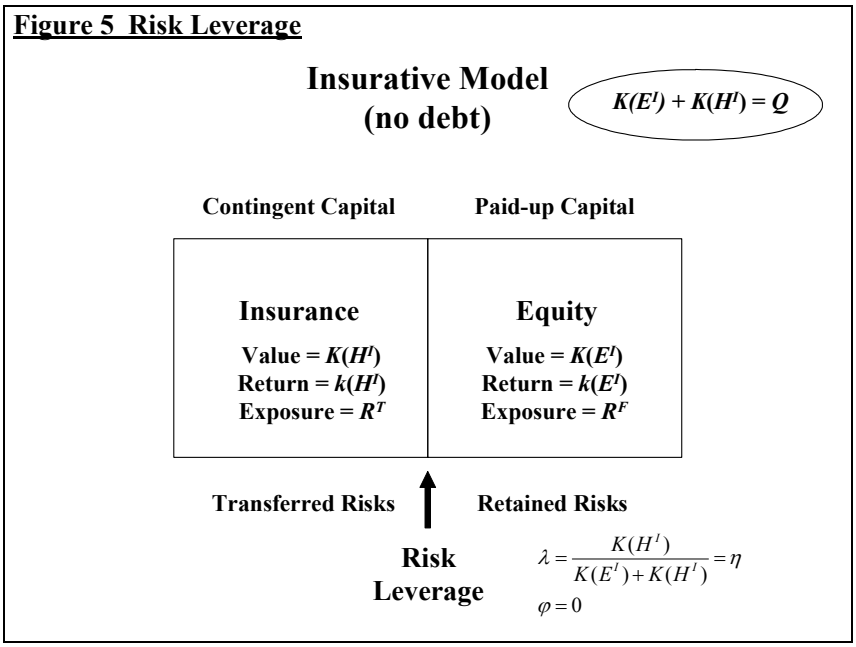
Equation 30 is the WACC equation stated earlier, with q equal to the WACC, w.

Figure 4 Financial Leverage



The shareholders, as owners of the firm and the underlying assets, have the right to do this. Their objective is to maximize return on equity by leveraging. The additional return over

the baseline return is not a free lunch; it is the result of changing the shareholders' risk profile. The amount of insurance premium is derived explicitly from these relationships.



Now consider the setting of the Insurative Model where baseline equity is replaced by a combination of equity, E^I, and insurance, H^I, but no debt (Figure 5). In this case, it is risk leverage, not financial leverage that provides the shareholders with the means to increase their return on equity.

Here, the firm's risks are specified as block that is stratified vertically into two columns representing retained and transferred risks. The retained risks are covered by the equity investors (there is no debt here) and the transferred risks are taken up by the insurers. The value of equity in this case is $K(E^I)$ and the equity value equivalent (EVE) of insurance is $K(H^I)$, so that $K(E^I) + K(H^I) = Q$. The return that the equity holders will get for the retained risk will be $k(E^I)$ appropriate for that risk. Similarly, the insurers will get a return of $k(H^I)$ on the transferred risk that they are covering. Note, as mentioned earlier, both $K(H^I)$ and $k(H^I)$ are measured from the perspective of the firm and not the insurer.

The shareholders will only insure if there is a *benefit* from risk leverage. There are two cases to consider here. First, assume that this is like the previous setup where the shareholders *increase their return* relative to q with risk leverage, i.e., $k(H^I) < q < k(E^I)$. As before, both the shareholders and the insurer are entitled to q as participants in the risks and

activities of the firm, except that the returns are modified to reflect the risk shifting between the two. This is summarized as below:

$$\begin{array}{lcccccc} \underline{\text{Risk taker}}: & & \underline{\text{As investor}} & & \underline{\text{Extra}} & & \underline{\text{Return}} \\ \text{Equity}: E^I \Rightarrow & & q & + & \{k(E^I)-q\} & = & k(E^I) \\ \text{Insurer}: H^I \Rightarrow & & q & - & \{q-k(H^I)\} & = & k(H^I) \end{array} \qquad (31)$$

The extra $\{q - k(H^I)\}$ reduction for the insurer is actually the benefit provided to the shareholder by the insurer for keeping the transferred risk. Again the two extra amounts look like insurance. Note however that although the equity holder is purchasing insurance, the relationship on returns is such that the equity holder shows an *increase* in return from insuring – there would be no reason to purchase insurance otherwise in this setting. In equilibrium, the two amounts paid and received must be equal.

$$K(E^I)\times\{k(E^I)-q\} = K(H^I)\times\{q-k(H^I)\}$$

from which (32)

$$q = k(E^I)\times\frac{K(E^I)}{K(E^I)+K(H^I)} + k(H^I)\times\frac{K(H^I)}{K(E^I)+K(H^I)}$$

Equation 32 is similar in form to WACC, but it is actually the TACC equation, with no debt. In this case the WACC equation does not apply. Conventional finance would argue that this firm is unlevered. The Insurative Model demonstrates that it is and that the TACC, τ, is q.

Unlike the case for WACC, the assumption that shareholders always increase their returns does not have to hold for risk leverage. The second case to consider is where risk transfer may be a benefit to shareholders if it removes some risks that they do not wish to carry, even if there is a *reduction in return* from q, i.e., $k(E^I) < q < k(H^I)$. Shareholders have a reduction of $\{q - k(E^I)\}$ and the insurer has an increase of $\{k(H^I) - q\}$. Equation 32 is still correct, with the premium equality holding true after a change of sign on both sides of the first line.

Finally, consider the setup of the Insurative Model, with baseline equity replaced by a combination of equity, E, debt D, and insurance, H (Figure 6). (We drop all superscripts on E, D and H here.) Having done the preceding, the analysis is quite straightforward. All the 3 forms of leverage are present, with the following values[10]:

$$\text{Financial Leverage}: \varphi = \frac{K(D)}{K(E)+K(D)}$$

$$\text{Insurative Leverage}: \eta = \frac{K(H)}{K(E)+K(D)+K(H)} \qquad (33)$$

$$\text{Risk Leverage}: \lambda = \frac{K(H)}{K(E)+K(D)+K(H)}$$

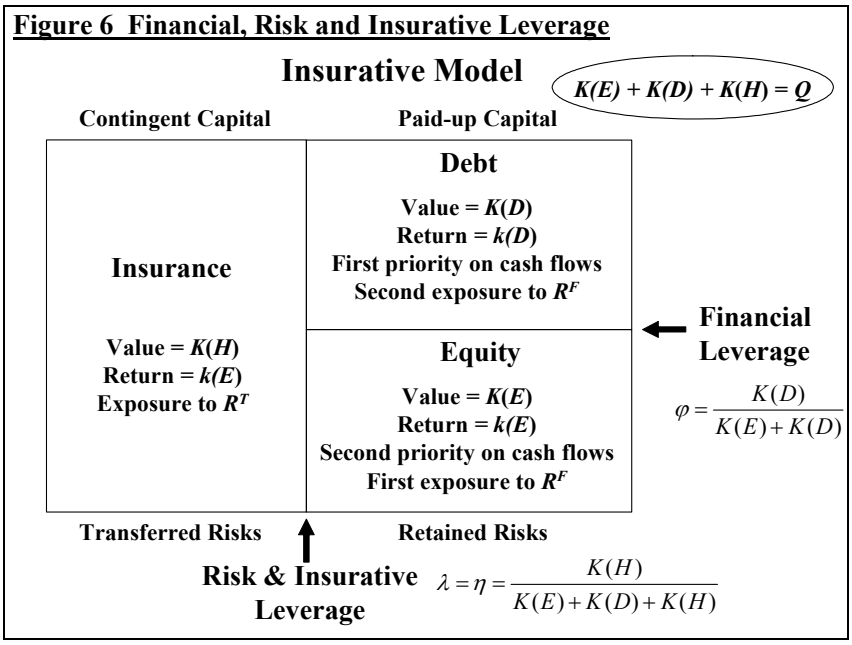
Combining Equations 30 and 32, we get the following TACC equation, which is the weighted average cost of paid-up capital (WACC) and contingent capital, and its relation to the leverage ratios:

$$\begin{aligned} q &= k(E)\times\frac{K(E)}{K} + k(D)\times\frac{K(D)}{K} + k(H)\times\frac{K(H)}{K} \\ &= \left\{k(E)\times\frac{K(E)}{K^P} + k(D)\times\frac{K(D)}{K^P}\right\}\times\frac{K^P}{K} + k(H)\times\frac{K(H)}{K} \\ &= \left\{k(E)\times\frac{K(E)}{K^P} + k(D)\times\varphi\right\}\times\frac{K^P}{K} + k(H)\times\eta \\ &= w\times\frac{K^P}{K} + k(H)\times\frac{K^C}{K} \end{aligned}$$

where (34)

$$\begin{aligned} K &= K(E)+K(D)+K(H) \\ &= Q \text{ when adequately capitalized} \\ K^P &= K(E)+K(D) \\ K^C &= K(H) \end{aligned}$$

Figure 6 Financial, Risk and Insurative Leverage



The Insurative Model leads to a natural view of risky debt being the sum of unleveraged equity investing plus an insurance policy (for which a premium is paid through the cost of debt) that gives debt holders a priority on assets. The analysis is also consistent with the idea in finance that in equilibrium, the risk-adjusted return to the equity holder must be the same, independent of leverage. The extra return to the shareholders from leverage must come from somewhere. This analysis shows that it comes from "insuring" the bondholders.

6 Summary

The Insurative Model can serve as a useful ERM framework through which risk management issues can be addressed consistently with other corporate finance decisions. Although we considered debt, equity and insurance separately, the framework is general enough to consider transactions that combine facets of all these instruments.

The framework has three important consequences:

1. Expanded forms of leverage:
   The Insurative Model shows that the conventional focus on financial leverage (i.e. the proportion of debt) is only one form of leverage. It identifies two other management actions that can impact the return to shareholders, Insurative leverage (proportion of contingent capital) and risk leverage (proportion of transferred risk).
2. Comprehensive measure of cost of capital:
   The Insurative Model recognizes that the popular conventional measure of capital cost, the weighted average cost of capital (WACC), is misspecified. It is actually the cost of paid-up capital and not of all capital resources. A better measure is the total average cost of capital (TACC), which includes all forms of capital.
3. Direct relationship between risk and capital management:
   The Insurative Model recognizes that the connection between risk and capital management may be more than just a direct one; it could be an identity. Analytically, if we consider risk management as an exercise in maximizing the return on the risks undertaken by a firm, subject to capital constraints, and capital management as an exercise in minimizing the cost of capital of a firm, subject to risk constraints, then this is equivalent to the following proposition (which will be the subject of another paper):

Risk Management and Capital Management are primal and dual versions of the same underlying optimization problem

One major challenge that the Insurative Model needs to address is the lack of data relating to some of the significant risks of corporations. (Clearly this is a challenge for other frameworks as well.) The framework needs to enable making decisions under uncertainty with sparse data, i.e. P5. This challenge can be addressed by relating it to a parallel problem in asset-liability management (ALM) and then incorporating the resulting approach in a dynamic Insurative Model setting.

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End Notes

[1] Henceforth the term "insurance" is used to mean all forms of financial contracts that transfer risk, such as insurance and hedging. It includes both indemnity contracts where protection can be purchased only if there is insurable interest, and hedging contracts where payments are made irrespective of whether a firm incurs a loss.

[2] A more reasonable expression for the last line of this equation assumes subadditivity for *f*, thereby replacing the equality sign, =, with the less-than-or-equal-to sign, #

[3] The same function, $f\{risk\}$, is used for this discussion to denote that there is a functional relationship between a risk and the capital required to cover it. The actual functional relationship between any particular component of firm risk and the capital required to cover it may be different.

[4] See Shimpi (1999) Chapter 9 for a full description of Contingent Capital.

[5] This definition relates Q and K only at the instant they are calculated. This definition is enough to describe the basics of the Insurative Model. Capital sufficiency and capital adequacy over time is beyond the scope of this paper.

[6] For indemnity contracts such as insurance, claims are paid only when the firm incurs a loss. In other contracts such as derivatives, the firm receives a payment even it does not incur a loss.

[7] ILS refers to insurance-linked securities such as catastrophe bonds.

[8] Although some finance academics have considered the effects of some off-balance-sheet capital on capital structure decisions, e.g. Merton and Perold (1993), there has not been uniform treatment in the literature.

[9] Strictly speaking, of course, the equity holders get relief from having to make up the shortfall to the bondholder.

[10] Although this stylized setting shows that risk leverage and Insurative leverage are the same, we have noted earlier that this is not usually the case.