Ratchet Equity Indexed Annuities

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Abstract

The Equity Indexed Annuity (EIA) contract offers a proportional participation in the return on a specified equity index, in addition to a guaranteed return on the single premium. The ratchet EIA design applies the participation rate to the equity index return separately in each year of the contract. In this paper we derive a valuation formula for the tractable, compound version of the ratchet EIA. We present a practical lattice method for valuing simple ratchet EIAs, which are not analytically tractable. We compare the accuracy of the lattice method with Monte Carlo simulation for the simple ratchet EIA and find that the lattice approach achieves a very high degree of accuracy much more efficiently than the Monte Carlo approach. The underlying life of contract guarantee value is estimated using Monte Carlo simulation; the value is shown to be very small compared with the ratchet part of the benefit.

KEYWORDS: OPTION PRICING; EQUITY Indexed ANNUITIES; MONTE CARLO

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1 Introduction

An Equity Indexed Annuity (EIA) contract is an important contract in the US insurance market. It provides a guaranteed annual return in combination with some participation in equity market appreciation. In the simplest form, it may be considered as a combination of a fixed interest deposit, together with a call option on an underlying risky asset, typically an equity price index such as the S&P 500\(^1\). Contract terms are short relative to other life insurance products, with seven years being typical. Participation in an equity index is offered by guaranteeing a specified proportion of the return on the index. This proportion is called the participation rate.

The options embedded in typical EIA contracts are often very similar to exotic options available in capital markets. Standard texts on exotic options, such as Zhang (1998), will give some useful background and formulae. The issue of valuing EIAs is discussed in a few papers; Lee (2002) develops formulae for barrier options, and Tiong(2001) uses Esscher transforms to value a contract slightly different to the usual contract design, under which the participation is applied to the log-return on the index, rather than the actual return. Lin and Tan (2003) look at the application of stochastic interest rates and mortality. All of these papers consider only contracts for which analytic solutions are available.

This paper will focus on one particular category of EIA contract, which we have called the ratchet EIA. Under the ratchet contract design, the participation in the equity index is applied separately in each year of the contract. There are two common types of ratchet EIA, the compound ratchet, where returns in each year are compounded, and the simple ratchet EIA where the returns in each year are added together to give the final payout. In both cases an underlying guaranteed interest rate applies to the total payout – we call this the ‘life of contract’ guarantee.

The compound ratchet option is analytically tractable, under the usual lognormal assumption for the equity index; the simple ratchet option and the life of contract guarantee are not. In this paper we develop a straightforward, accurate numerical method for valuing the simple ratchet option. We derive the formula for the compound ratchet option, as it will help us to assess the accuracy of the simple ratchet valuation. We also explore the life of contract guarantee value, and the practice of averaging the index before applying the participation rate. The objective of this paper is to provide a comprehensive, practical

\(^1\)Standard and Poors
guide to ratchet EIA valuation. Some of the results are not new; the valuation formula in equation 4 has been derived elsewhere, in particular in Boyle and Tan (2002) and Hardy (2003), and similar equations are found in Tiong (2001) and Lee (2002). The final section which discusses averaging the index contains material well known in the field of option theory, but not so well known to actuaries. The purpose of putting all this material in this paper is to provide a reasonably complete picture of the issues and techniques that are important in the valuation of ratchet options, designed for practical implementation by actuaries involved in ratchet Equity-Indexed Annuities.

At around seven years, the typical EIA contract term is a lot shorter than the 20-30 years common for a variable annuity, but it is still a long term for an option. In the market, the cost of the option would include a very substantial margin for uncertainty. The effect of mortality on EIAs is slight, and we have not allowed for it in this paper. It has been well documented, for example in Boyle and Schwartz (1977) and Lin and Tan (2003) that the appropriate method for managing the mortality effect is deterministically – that is, to multiply payoffs by the appropriate survival or mortality probabilities. We have also assumed constant interest rates. See Lin and Tan (2003) for some exploration of the effect of assuming stochastic interest rates.

The remainder of this paper is organized as follows:

- we derive the analytical form for the payoff from the compound ratchet EIA design, in Sections 2 and 3;
- we demonstrate how the non-recombining trinomial lattice provides a very accurate estimate of the value of the compound ratchet EIA design in Section 4;
- also in Section 4 we demonstrate the use of the non-recombining trinomial lattice for valuing the simple ratchet EIA design;
- in Section 5 we apply the more common method of valuing analytically non-tractable guarantees such as the simple ratchet option – using Monte Carlo simulation. We use the compound ratchet option as a control variate to reduce the standard error of the estimate of the simple ratchet option value;
- we explore the problem of the life-of-contract guarantee in Sections 4 and 5.
Under the annual ratchet method, the index participation is evaluated year by year. Each year the payout figure is increased by the greater of the ‘floor’ rate – usually 0%, and the increase in the underlying index, multiplied by the participation rate. The increases may be simple or compound. A common variant is to average the index over some period (for example one year). In the following we assume that the equity index is used without any averaging. Also, we ignore mortality here as the effect for these short contracts is not great. In fact, it is straightforward to incorporate mortality if required, but omitting it here makes the derivation of the option formulae easier to follow.

We will express the ratchet payoff symbolically. Let $f$ be the floor rate, $c$ the ceiling rate, $S_t$ the index at $t$ and $\alpha$ the participation rate. The term is $n$ years, and the ratchet is applied annually. The initial investment is $P$, of which $P(1 - k)$ is deducted at issue for expenses and risk premium. Typically $k$ ranges from 87% to 95%. The guarantee rate is $g$, typically 3% per year, effective. Then the payout under a compound annual ratchet contract, without averaging, is

$$B = \max (Pk(1 + g)^n, CRP) = CRP + \max (Pk(1 + g)^n - CRP, 0)$$

where $CRP = \prod_{t=1}^{n} 1 + \min \left( \max \left( \alpha \left( \frac{S_t}{S_{t-1}} - 1 \right), f \right), c \right)$

The simple annual ratchet contract is similar; in this case

$$B = \max (Pk(1 + g)^n, SRP) = SRP + \max (Pk(1 + g)^n - SRP, 0)$$

where $SRP = 1 + \sum_{t=1}^{n} \min \left( \max \left( \alpha \left( \frac{S_t}{S_{t-1}} - 1 \right), f \right), c \right)$

Now the payoff under the ratchet EIA is an option on an option. The ratcheted premium, $CRP$ or $SRP$ is itself an option, and is also the underlying risky asset for the EIA payoff, which we can think of as the sum of the ratcheted premium, and a put option $\max (Pk(1 + g)^n - CRP, 0)$ or $\max (Pk(1 + g)^n - SRP, 0)$

The valuation of the ratchet option under standard Black-Scholes assumptions can be done exactly for the compound case, $CRP$, but requires numerical methods for the simple ratchet, $SRP$. In both cases the additional put option, applying the flat $g$ per year
guarantee, requires numerical methods. We call this additional guarantee the ‘life of contract’ guarantee.

3 Valuation of the compound ratcheted premium

To find the value of the ratcheted premium using Black-Scholes assumptions we take expectation under the risk neutral distribution of the payout, discounted at the risk free rate of return. We use here the standard lognormal model for the equity index, $S_t$, so that the (non-overlapping) accumulation factors $S_t/S_{t-1}$ are independent and identically lognormally distributed. We must allow for the fact that participation is applied to the price index only, and dividends are not distributed. This is easily allowed for in the Black-Scholes-Merton option framework by adjusting the parameters of the lognormal distribution to $r - d - \sigma^2/2$ and $\sigma^2$ in the risk neutral distribution, where $d$ is the continuously compounded rate of dividend per year.

The value of the $n$-year CRP is then

$$E[e^{-rn} CRP] = E \left[ e^{-rn} \prod_{t=1}^{n} \left( 1 + \min \left( \max \left( \alpha \left( \frac{S_t}{S_{t-1}} - 1 \right), f \right), c \right) \right) \right]$$

$$= \prod_{t=1}^{n} E \left[ e^{-r} \left( 1 + \min \left( \max \left( \alpha \left( \frac{S_t}{S_{t-1}} - 1 \right), f \right), c \right) \right) \right]$$

Under the risk neutral distribution, $R_t = S_t/S_{t-1}$ is lognormally distributed, and

$$\left( 1 + \min \left( \max \left( \alpha (R_t - 1), f \right), c \right) \right) = \begin{cases} 1 + f & \text{when } R_t < 1 + \frac{f}{\alpha} \\ 1 - \alpha + \alpha R_t & \text{when } 1 + \frac{f}{\alpha} < R_t \leq 1 + \frac{c}{\alpha} \\ 1 + c & \text{when } R_t > 1 + \frac{c}{\alpha} \end{cases}$$

Let $K_1 = 1 + \frac{f}{\alpha}$, $K_2 = 1 + \frac{c}{\alpha}$. Then applying the lognormal distribution, with pdf $f_R(x)$ we have

$$E \left[ e^{-r} \left( 1 + \min \left( \max \left( \alpha (R_t - 1), f \right), c \right) \right) \right]$$

$$= \int_0^{K_1} e^{-r} (1 + f) f_R(x) dx + \int_{K_1}^{K_2} e^{-r} (1 - \alpha + \alpha x) f_R(x) dx + \int_{K_2}^{\infty} e^{-r} (1 + c) f_R(x) dx$$

$$= e^{-r} (1 + f) \Phi(-d_2) + e^{-r} (1 - \alpha) \left( \Phi(d_2) - \Phi(d_4) \right)$$
where $\Phi()$ is the standard normal distribution function and

\[
d_1 = \frac{\log \frac{1}{K_1} + r - d + \sigma^2/2}{\sigma} \quad \text{and} \quad d_2 = d_1 - \sigma
\]

\[
d_3 = \frac{\log \frac{1}{K_2} + r - d + \sigma^2/2}{\sigma} \quad \text{and} \quad d_4 = d_3 - \sigma
\]

The CRP is therefore

\[
CRP = \left\{ e^{-r}(1 + f)\Phi(-d_2) + e^{-r}(1 - \alpha) (\Phi(d_2) - \Phi(d_4)) + \alpha e^{-d} (\Phi(d_1) - \Phi(d_3)) + e^{-r}(1 + c)\Phi(d_4) \right\}^n
\]

(4)

So, for example, suppose we have a compound ratchet EIA with $100 premium. Table 1 gives the value of the compound ratchet premium for various combinations of cap rate, $c$, and participation rate $\alpha$. We will repeat this set of examples in subsequent sections. The pertinent assumptions other than the cap and participation rates are:

- Seven year ratchet EIA contract;
- 25% per year volatility;
- 6% per year risk free rate of interest, continuously compounded;
- 2% per year dividend yield on the index, continuously compounded;
- Floor rate of interest, $f = 0\%$.

So, if the only benefit is a $100 ratcheted premium, with, for example, 100% participation in the equity (price only) index, with a cap of 15% and a floor of 0%, then the price is $99.00$.

The life of contract guarantee provides, typically, an annual return of 3% on 90% of the initial premium, giving a minimum payment at maturity of $110.69 per $100 premium. This is greater than the minimum payment under the compound ratchet, which is $100 (where the return each year falls below the 0% floor). The additional cost of the life of
## Table 1: Price of the compound ratcheted premium, % of premium.

<table>
<thead>
<tr>
<th>Participation (α)</th>
<th>Cap rate c (per year)</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>30%</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td></td>
<td>85.94</td>
<td>93.01</td>
<td>98.15</td>
<td>104.11</td>
<td>108.24</td>
</tr>
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<td>0.8</td>
<td></td>
<td>87.60</td>
<td>96.60</td>
<td>104.04</td>
<td>114.57</td>
<td>126.87</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td>88.66</td>
<td>99.00</td>
<td>108.22</td>
<td>122.89</td>
<td>148.18</td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td>89.39</td>
<td>100.71</td>
<td>111.29</td>
<td>129.51</td>
<td>172.49</td>
</tr>
</tbody>
</table>

contract guarantee is not suitable to evaluate analytically. We cannot, for example, simply replace the floor value for the ratchet with 3%, since this would substantially over-estimate the value of the life of contract guarantee. Compare, for example, Contract A with 0% per year floor and 3% per year life of contract guarantee with Contract B which has a 3% floor and no life of contract guarantee. Suppose both contracts have a participation rate of 80% and a cap of 15%. Both have a 4 year term. Now suppose that the returns over the 4 years are 25%, -10%, -10%, 10%. Then the ratchet part of Contract A pays out $100(1.15)(1.0)(1.0)(1.08) = 124.2$. This is greater than the life of contract minimum, $90(1.03)^4 = 101.30$, so the 3% guarantee does not enter into the calculation.

Under Contract B, the payout is $100(1.15)(1.03)(1.03)(1.08) = 131.76$ which is much greater than the payout under Contract A. Clearly, we cannot use Contract B as a short cut to the value for the Contract A payoff. The addition of the life of contract guarantee to the compound ratchet premium requires numerical methods for valuation.

The simple ratchet option value involves the sum of lognormal random variables, which is not analytically tractable. Simple ratchet contracts are therefore valued using numerical methods.

The most common approach for determining the value of the life of contract guarantee and the simple ratchet option might be Monte Carlo simulation. Any European option can be valued using Monte Carlo simulation, by projecting the present value of the option payoff under the risk neutral distribution, and taking the mean value. With a sufficiently large number of simulations, we can get arbitrarily close to the true option value. However, this is not necessarily the most efficient method for estimating the option price. In this paper we propose a trinomial lattice method, which is well suited to the format of the simple ratchet problem.
4 The non-recombining trinomial lattice

Each year under the ratchet EIA option, the interest credited is either the floor rate, the cap rate, or some rate between the cap and floor, $\alpha(R_t - 1)$. Under the trinomial lattice, we approximate this process by assuming that there are only three outcomes; the cap, the floor, or the mean credited rate given that the rate falls between the cap and the floor. The probabilities associated with each outcome each year are calculated using the risk neutral distribution. Boyle and Tan (2001) show how a recombining trinomial lattice can be used for the valuation. In this paper, we use a non-recombining lattice, as the results are significantly better, at a small cost in terms of additional complexity.

Each year the credited rate is the floor rate, $f$, if $\alpha(R_t - 1) \leq f$, with probability

$$
\Pr \left[ R_t \leq 1 + \frac{f}{\alpha} \right] = \Pr[R_t < K_1] = \Phi \left( \frac{\log K_1 - (r - d - \sigma^2/2)}{\sigma} \right) = \Phi(-d_2)
$$

Similarly, the credited rate is $c$ if $\alpha(R_t - 1) > c$ with probability $\Phi(d_4)$.

The probability that the credited rate falls between the floor and the ceiling then must be $1 - \Phi(-d_2) - \Phi(d_4) = \Phi(d_2) - \Phi(d_4)$. In this case, under the trinomial lattice method, we assume the credited rate is $m$ say, where

$$
E \left[ \alpha(R_t - 1) \mid R_t > K_1 \text{ and } R_t \leq K_2 \right] = \frac{\alpha}{\Phi(d_2) - \Phi(d_4)} \int_{K_1}^{K_2} (y - 1) f_R(y) dy \quad \text{where } f_R \text{ is the lognormal pdf}
$$

$$
= \alpha \left( e^{r-d} \frac{\Phi(d_1) - \Phi(d_3)}{\Phi(d_2) - \Phi(d_4)} - 1 \right)
$$

The lattice is non-recombining, in the sense that the cap rate followed by the floor rate does not equal the central rate. However, the order of the ratchet rates does not matter, so that we do not have a lattice of $3^n$ nodes for an $n$-year contract. The outcome at the contract maturity is the same for any combination where, for example, the cap rate applies
n_c years, the floor rate applies n_m years and the middle rate applies in n - n_c - n_m years, regardless of the order in which the rates apply. By using a non-recombining lattice, we can set the middle rate more accurately, at the expense of extra nodes at the maturity date.

For example, for a 7-year contract, there are 36 possible outcomes, combining the cap, floor and middle rates. We apply a multinomial probability distribution where the parameters are the number of years, and the probabilities associated with the cap, floor and middle rates. That is, let (n_f, n_m, n_c) denote an outcome arising from a path where the floor rate arises n_f times, the middle rate n_m times, and the cap rate n_c times, where 0 ≤ n_f, n_m, n_c ≤ 7 and n_f + n_m + n_c = 7. Let p_f, p_m and p_c be the associated probabilities \( \Phi(-d_2), \Phi(d_2) - \Phi(d_4) \) and \( \Phi(d_4) \) respectively. The probability associated with each path through the tree described by \( (n_f, n_m, n_c) \) is

\[
\frac{7!}{n_f! n_m! n_c!} p_f^{n_f} p_m^{n_m} p_c^{n_c}
\]

and the payout associated with each outcome \( (n_f, n_m, n_c) \) for the compound ratchet with guaranteed minimum \( G \) is

\[
\max(G, P(1 + f)^{n_f} (1 + m)^{n_m} (1 + c)^{n_c})
\]

and for the simple ratchet with guaranteed minimum \( G \) the payout is

\[
\max(G, P (1 + n_f f + n_m m + n_c c))
\]

The 36 possible payouts for the compound or simple ratchet option are easily programmable in Excel.

The lattice is remarkably accurate for valuing the compound ratchet premium, without the life of contract guarantee. For example, using the lattice for some of the values in table 1, we get values different from that table by a factor of around \( 10^{-12} \) (by Excel accuracy), even where the cap is so large as to be negligible, in which case we effectively have a binomial lattice. Of course, since we can calculate the compound ratchet premium value analytically, the accuracy of the lattice is not very useful.

More helpfully, the lattice appears to work well also for the simple ratchet premium. In Table 2 we show estimated values for the simple ratchet premium, corresponding to Table 1 for the compound premium. In the next section we will re-estimate these values using
<table>
<thead>
<tr>
<th>Participation</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>30%</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>83.6851</td>
<td>89.1147</td>
<td>92.8456</td>
<td>96.9644</td>
<td>99.7032</td>
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<td>103.7266</td>
<td>111.0361</td>
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<tr>
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<td>85.8197</td>
<td>93.4475</td>
<td>99.6854</td>
<td>108.7400</td>
<td>122.3689</td>
</tr>
<tr>
<td>1.2</td>
<td>86.3831</td>
<td>94.6418</td>
<td>101.6655</td>
<td>112.5247</td>
<td>133.7017</td>
</tr>
</tbody>
</table>

Table 2: Price of the simple ratcheted premium, % of premium.

Monte Carlo simulation, to assess the accuracy and efficiency of the trinomial method.

The life of contract guarantee may be estimated by replacing the payout under the ratchet with the guaranteed payout whenever the ratchet payout is smaller. This gives fair results for the life of contract guarantee when the cap is small, by which we mean that the cap rate accumulation factor is not too far from the guaranteed accumulation factor, but it gives relatively very poor estimates of the value, in general. The problem with the lattice is that, while we are using 36 points to estimate the value of the option as a whole, only perhaps 1 or 2 points are involved in the valuation of the life of contract guarantee. We can get a very good estimate of the mean outcome, but the lattice is not accurate for values in the tails.

5 Monte Carlo simulation

5.1 Simple Ratchet Premium

In this section we develop estimates for the simple ratchet premium using Monte Carlo simulation. This is not new, we do this to assess the accuracy of the lattice method.

Using Monte Carlo simulation we can generate paths for the stock price process $S_t$ under the risk neutral distribution, and then determine the payoff for each path. The estimated price of the EIA is the mean of the payoffs, discounted at the risk free rate.

It is straightforward to use the control variate method of variance reduction for this contract, which will reduce the sampling error associated with the estimate. We use the compound ratcheted premium as the control variate, as it can be calculated analytically.
<table>
<thead>
<tr>
<th>Participation α</th>
<th>Cap rate c (per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>0.6</td>
<td>83.6853</td>
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<td>1.2</td>
<td>86.3830</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Table 3: Price of the simple ratcheted premium, % of premium, using Monte Carlo simulation, $10^7$ simulations.

<table>
<thead>
<tr>
<th>Participation α</th>
<th>Cap rate c (per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.0002</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0002</td>
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<tr>
<td>1</td>
<td>-0.0001</td>
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<tr>
<td>1.2</td>
<td>0.0001</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Table 4: Difference between Monte Carlo estimate and Lattice estimate of simple ratchet premium value, % of premium.

This is then used to calibrate the set of scenarios used in the Monte Carlo simulation for the EIA values that are not analytically tractable. For more information on the use of control variates see, for example, Hardy (2003) or Boyle et al (1997).

Comparison of Tables 2 and 3 show that the lattice does a very decent job of estimating the simple ratchet premium values. In Table 4 we show the differences between the trinomial lattice estimation and the Monte Carlo estimation; we include the Monte Carlo standard errors for comparison. The maximum difference is less than 0.01% of the premium, and the lattice estimate is within around one standard error of the Monte Carlo estimate for all entries except two.

In summary, the lattice appears to be a very practical way of valuing the simple premium contract; more efficient than simulation, with accuracy comparable to $10^7$ or more simulations.
### Compound Ratchet

<table>
<thead>
<tr>
<th>Participation</th>
<th>Cap rate $c$ (per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>$\alpha$ 0.6</td>
<td>0.203</td>
</tr>
<tr>
<td>$\alpha$ 0.8</td>
<td>0.158</td>
</tr>
<tr>
<td>$\alpha$ 1.0</td>
<td>0.134</td>
</tr>
<tr>
<td>$\alpha$ 1.2</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Standard errors range from 0.0007 to 0.001.

### Simple Ratchet

<table>
<thead>
<tr>
<th>Participation</th>
<th>Cap Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>$\alpha$ 0.6</td>
<td>0.204</td>
</tr>
<tr>
<td>$\alpha$ 0.8</td>
<td>0.159</td>
</tr>
<tr>
<td>$\alpha$ 1.0</td>
<td>0.135</td>
</tr>
<tr>
<td>$\alpha$ 1.2</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Standard errors range from 0.0007 to 0.001.

Table 5: Estimated cost of the Life of Contract guarantee (90% of premium, 3% per year) using Monte Carlo simulation, % of premium.

### 5.2 Life of Contract Guarantee

For the Life of Contract guarantee we have used standard Monte Carlo Simulation without variance reduction. The results of $10^6$ simulations are summarized in Table 5. The numbers here are the additional cost of the life of contract guarantee in addition to the compound or simple ratchet option. From this table we notice that:

1. The cost of the life of contract guarantee is very small, at a maximum of around $0.20 for a $100.00 premium.

2. The life of contract guarantee is very similar in cost for the simple and compound versions of the ratchet EIA. This is because most of the cost arises when the ratchet premium proceeds are at their minimum – in both cases, the minimum ratchet payment is 1.0.
6 Conclusion

The main contribution of this paper is to demonstrate that the simple ratchet premium EIA can be valued very efficiently and accurately using the non-recombining trinomial lattice approach. The accuracy compares with a Monte Carlo approach using a control variate, and $10^7$ simulations, but the lattice method is clearly much more efficient.

We have also shown that the compound ratchet premium can be valued analytically using the usual lognormal risk neutral distribution.

The underlying life of contract guarantee is not analytically tractable, but the value is small, and is not very sensitive to the cap or participation rates, nor to whether the ratchet is simple or compound.

All the valuation in the paper has assumed a standard lognormal risk neutral distribution. Because the ratcheting is annual, the monthly or daily correlations between equity returns are not important here. Annual returns on equities have insignificant autocorrelation for most well used indices, so the serial independence assumption is reasonable. In addition, we are concerned largely with the center of the equity return distribution, which will be less susceptible to model error than tail measures.

References


