Intergenerational transfers within funded pension schemes*

Jiajia Cui† Frank de Jong‡ Eduard Ponds§
University of Amsterdam & ABP Pension Fund

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Abstract

In this paper we model the transfers of value between the various generations in a funded pension scheme. Value calculaton is based on contingent claims valuation (deflator method). We define a setting like in the Netherlands, where pension benefits and contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). Furthermore, we define explicit risk allocation rules, specifying who of the stakeholders, when, and to what extent is taking part in risk-bearing. Such a pension deal may lead to substantial transfers of wealth between young and old generations, depending on the asset returns and the exact risk allocation scheme. Using contingent claims valuation method, we calculate these transfers and the distribution of value across generations. The pension schemes that provide safer and smoother consumption streams are ranked higher in utility terms. The smoother consumption stream can be achieved by allowing risk shifting over time, using mainly indexation instruments instead of contribution instruments.

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†University of Amsterdam, ABP Pension Fund and Tinbergen Institute, Roetersstraat 31, 1018 WB Amsterdam, The Netherlands. E-mail: cui@tinbergen.nl

‡University of Amsterdam, Finance Group, Faculty of Economics and Econometrics, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands. E-mail: fdejong@uva.nl

§ABP Pension Fund and University of Amsterdam. E-mail: e.ponds@abp.nl

1 Introduction

Funding decisions in pension funds in the past have been dominated by the traditional actuarial approach. The main goal of this approach is to arrive at stability in the course of the contribution rate and the funding ratio over time. The approach is typically grounded on rules of thumb as to valuation and accounting issues. The actuarial approach recently has been heavily criticized (see for example Exley *et al.* 1997, Bader&Gold 2002, Chapman *et al.* 2001). The approach contrasts sharply with the worldwide trend in accounting standards towards more transparency through market-based reporting based on fair value. The fair value approach is based on economic principles. The application of economic principles makes possible to restate funding issues of pension funds in terms of 'economic value'. Economic value implies risk-adjusted valuation of future outcomes. This paper applies the value-based approach for pension fund issues, in particular regarding the issue of intergenerational transfers of value.

Chapman *et al.* (2001) employ the value-based approach to strategic decision making within a *company pension fund*. They model the fund not as a self-contained fund but simultaneously alongside the company. The stakes of the various parties aggregate to 100% of the assets of the company, including the assets of the pension scheme. Strategic decisions as to the investments, contribution rate and indexation have no impact on the total economic value of the combined stakes of the stakeholders. However, these decisions may well lead to transfers of value between the stakeholders. How and to what extent will depend on rules concerning the allocation of a funding shortage or funding surplus. Chapman et al. (2001) investigate different allocation rules and make clear that the pension fund must be seen as a zero-sum game. The value-based approach clarifies who gains and who loses from changes in strategic policy variables.

We employ the value-based approach for a pension fund based on intergenerational risk-sharing. This kind of risk-sharing can be found in public sector pension funds and industry pension funds in countries as UK, USA, Canada and the Netherlands. We present a framework to model the transfers of value between the various generations in a pension fund. Members of the fund pay contributions during their working life and receive defined pension benefits after retirement. Pension benefits and contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). We define explicit risk allocation rules, specifying who of the stakeholders, when, and to what extent is taking part in risk-bearing. Such a pension deal may lead to substantial transfers of wealth between young and old generations,

depending on the asset returns and the exact risk allocation scheme. Using contingent claims valuation methods, we calculate these transfers and the distribution of value across generations for alternative sets of risk-allocation. In total, we distinguish six stylized, distinct variants in funded plans based on collective risk-bearing. The 'real life' pension deals as performed by pension funds can be composed of components of these stylized variants. The alternative pension deals have no impact on the total economic value of the combined stakes of the stakeholders, however they will lead to diffirent outcomes as to the distribution of total value amongst the stakeholders. We also show that pension deals with safer and smoother consumption streams are ranked higher in utility terms. The smoother consumption stream can be achieved by allowing risk shifting over time, using mainly indexation instruments instead of contribution instruments.

The paper builds on previous contributions in this field, such as Blake (1998), Chapman et al. (2001), Ponds (2003), and De Jong (2003).

2 Pension Deals and Risk Allocation

2.1 Dynamics of the pension fund balance sheet

The fund has a stream of real pension promises, b_s, projected real premium income, p_s, for all $s > t$. The present value of the stream of benefits and premiums is determined by the real interest rate r. For simplicity, we assume this real interest rate is non-stochastic and valid for all maturities, so we have a flat term structure of real rates. The present values of the promised benefits and premiums is

$$PVb_t = \int_t^\infty e^{-r(s-t)} b_s ds \tag{1}$$

$$PVp_t = \int_t^\infty e^{-r(s-t)} p_s ds \tag{2}$$

In nominal terms, the present values should be multiplied by the price level Π_t. The price level in each year is given by the cumulative inflation π_t

$$\Pi_t = \exp\left\{\int_0^t \pi_s ds\right\} \tag{3}$$

The nominal values of the target premiums and benefits are where $B_t^* = \Pi_t b_t$ and $P_t^* = \Pi_t p_t$, and the present values are $PVB_t = \Pi_t PVb_t$ and $PVP_t =$

$\Pi_t PVp_t$. These values are not fixed but change with time and shocks in the price level. Using differentiation we find

$$\begin{aligned} dPVB_t &= [-B_t^* + (r+\pi_t)PVB_t]dt & (4) \\ dPVP_t &= [-P_t^* + (r+\pi_t)PVP_t]dt & (5) \end{aligned}$$

The surplus of the fund is defined as the value of its assets, plus the present value of future premiums, minus the present value of promised benefits

$$S_t = A_t + PVP_t - PVB_t \tag{6}$$

Each period, the fund receives premiums pays out benefits. These actual payouts may be different from the promised benefits and premiums, for example because they are determined by a policy ladder. In the remainder of this note, we will assume that we can write the net cash flow (actual premiums received minus benefits paid) as a function of the current surplus, and the target benefits and premiums:

$$B_t = f(S_t, B_t^*), \qquad P_t = f(S_t, P_t^*) \tag{7}$$

The dynamics of the surplus are given by the sum of the cash in and outflows and the change in value of assets and the present values of premiums and benefits

$$dS_t = (P_t - B_t)dt + dA_t + dPVP_t - dPVB_t \tag{8}$$

Substituting the various definitions, we find

$$dS_t = [(P_t - P_t^*) - (B_t^* - B_t) + R_t A_t - (r+\pi_t)(PVB_t - PVP_t)]dt \tag{9}$$

where R_t denotes the asset return. This equation is very intuitive: the change in the fund's surplus is determined by the difference between expected and actual cash flows, the asset returns, minus the increase in present value of (net) promised future benefits due to inflation and the cost of capital (the real interest rate). In discrete time, the surplus evolves as

$$S_{t+1} = S_t + (P_t - P_t^*) - (B_t^* - B_t) + R_t A_t - (r+\pi_{t+1})(PVB_t - PVP_t) \tag{10}$$

An interesting special case is the steady state, where real premiums and promised benefits are fixed values, i.e. $b_t = b$ and $p_t = p$. In that case, the real present values of benefits and premiums are time-invariant, given by

$PVb = b/r$ and $PVp = p/r$. The nominal present values dynamics then are only affected by the inflation

$$dPVB_t = \pi_t PVB_t dt, \quad dPVP_t = \pi_t PVP_t dt \tag{11}$$

and the surplus dynamics become

$$dS_t = (P_t - B_t)dt + R_t A_t - \pi_t(PVB_t - PVP_t)dt \tag{12}$$

This equation is also very intuitive: the change in the fund's surplus is determined by the net cash flow, the asset returns, and the effect of inflation on the present value of (net) promised future benefits. The discrete time equivalent is

$$S_{t+1} = S_t + P_t - B_t + R_t A_t - \pi_{t+1}(PVB_t - PVP_t) \tag{13}$$

2.2 Benefit obligations and funding

The previous equations are derived for a going-concern pension fund, so S_t is a construct determined by present values of ongoing streams. However this constructed surplus will also be the actual surplus when the present value of contributions matches the present value of new accrued benefits. This can be easily seen by decomposing the benefit obligations in Accrued Benefit Obligations, ABO, being the present value of the accrued benefits in the past up to present t. The second component are the the additional benefits to be accrued in the future by current participants as well as new participants, which we denote by $newBO$.

$$PVB_t = ABO_t + newBO_t \tag{14}$$

When the fund charges a 'costprice' contribution rate for future benefits, the present value of premiums equals the value of new benefit obligations, $PVP_t = newBO_t$. The actual surplus is defined as the difference between the assets A_t and the present value of accrued benefits in the past up to the present t, denoted by the term ABO_t:

$$S_t = A_t - ABO_t \tag{15}$$

In that case, the actual funding residual S_t will also be the surplus for a going-concern balance sheet as long as the present value of contributions matches the present value of new accrued benefits, irrespective of the length of the horizon.

2.3 Pension deals

Below we discuss some pension deals. Each of them may be seen as a 'corner' with respect to the aspect of risk bearing. We discuss the following deals:

Deal	Plan design	Risk shifting over time	Risk allocation
1	DB	No	Current contributions
2	DB	Yes	Projected contributions
3	Collective DC	No	Accrued benefits
4	Collective DC	Yes	Projected benefits
5+6	Hybrid	Yes	Projected benefits and contributions

Risk-shifting over time does mean that a funding residu in any period may be shifted forward to the future.

Deal 1: Defined Benefit scheme

In a pure Defined Benefit (DB) plan, benefits always will be paid out according to the promise. These benefits are real of nature, so every increase in the price level π_t leads to an equal increase in promised benefits:

$$i_t = \pi_t \tag{16}$$

where i_t denotes the indexation rate applied to all accrued benefits. The risk in the funding process is borne by the active generations. The contribution in period t consists of two parts. First, workers pay the 'costprice' P_t^* to fund new accrued benefits. This contribution is solved from the equality: $PVP_t^* = newBO_t$. Secondly, workers pay an additional contribution in order to restore the balance sheet of the pension fund, P_t^{add}. Hence, total contribution in period t, P_t, is equal to:

$$P_t = P_t^* + P_t^{add} \tag{17}$$

In this corner variant of a DB plan, the additional contribution have to absorb any funding surplus or deficit within one period in order to realize a full funded situation at the end of the period:

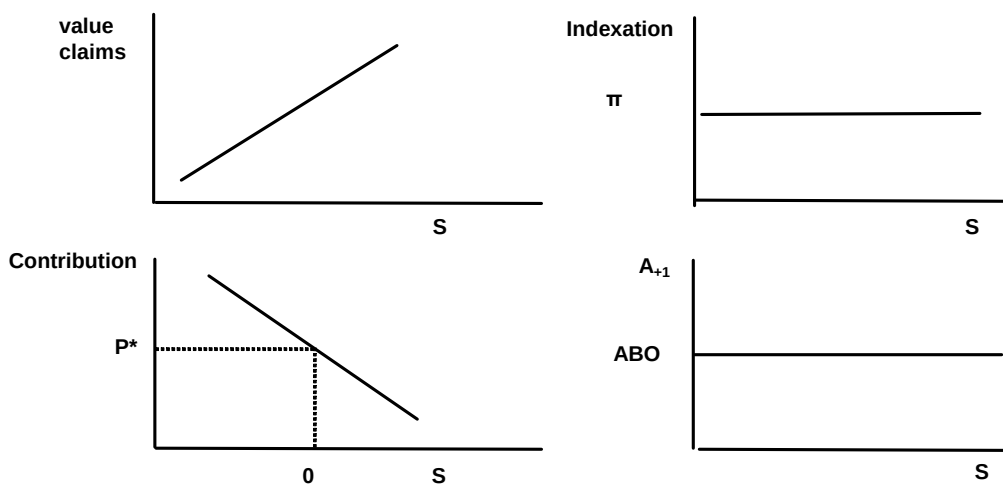
$$P_t^{add} = -S_t \tag{18}$$

Next period assets therefore are equal to the present value of accrued benefits at the end of period t:

$$A_{t+1} = ABO_t \tag{19}$$

Figure 1 below displays the relationship between the relevant variables with the actual surplus each period:

Figure 1: Pure Defined Benefit Plan



2.3.1 Deal 2: Defined Benefit scheme with risk shifting in time

Deal 1 will imply a very volatile additional contribution P_t^{add} because in every period this term absorbs the shocks in the rate of return and in the inflation rate.

Now let us assume that it is allowed to spread risk over time. The allowed horizon is set equal to the horizon used to calculate the costprice contribution. This assumption allows us to relate the additional contribution to the costprice contribution:

$$P_t^{add} = \frac{-S_t}{PVP_t^*} P_t^* \tag{20}$$

The dynamics in the surplus or in the assets of the pension fund has to be adjusted accordingly as follows:

$$A_{t+1} = ABO_t + S_t + P_t^{add} \tag{21}$$

For *a stationary scheme*, the costprice contribution is constant over time[1]. If so, then expression (20) can be reduced to:

$$P_t^{add} = -rS_t \tag{22}$$

Using $P_t^* = \Pi_t p$ and $PVP_t^* = \Pi_t PVp = \Pi_t p/r$, we get $P_t^{add} = \frac{-S_t}{PVP_t^*} P_t^* = -rS_t$. When there are no further shocks, deal 2 implies that the residual will not shrink because the additional contribution exactly matches the interest load of an undistributed surplus or of the uncovered deficit. A decrease in the residual will only occur when: $|P_t^{add}| > |-rS_t|$.

In a more general form, we can scale up expression (20) by a factor α, which lies between 1 and $1/r$ so that respectively Deal 1 and Deal 2 are included as two extreme cases:

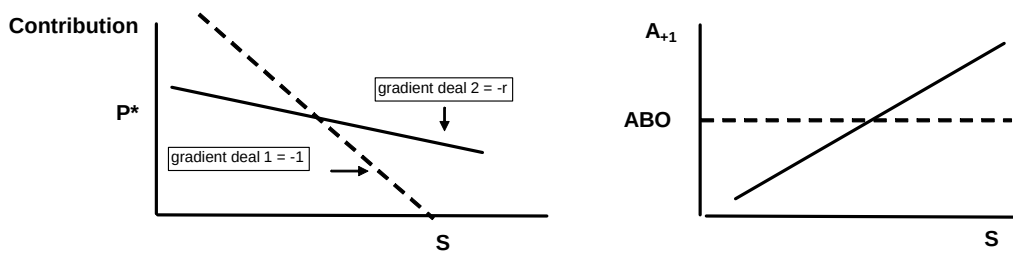
$$P_t^{add} = -\alpha r S_t \qquad 1 \leq \alpha \leq \frac{1}{r} \tag{23}$$

An additional contribution scheme like (23) is designed to achieve smoothing in the path of the additional contribution rate over time. The smoothing arrived at $\alpha = 1$ as in deal 2 is the highest one can reach. In Deal 1 ($\alpha = 1/r$) there is no smoothing at all. So, Deal 1 and Deal 2 are two corner solutions for the additional contribution, which is depicted in Figure 2. Deal 1 is depicted by the dashed line with a slope of -1. Deal 2 is represented by the solid line with a slope of $-r$. The between solutions are infinitely many, all with a slope of anything between $-r$ and -1 depending on how fast a funding residu have to be absorbed. The following table gives some examples of the half-life of absorbing the funding residu, i.e. the time taken for the surplus to reach its half level, for some chosen α values. In our analysis in Section 3, we set $\alpha = 2$ to characterize a deal with a long risk-shifting period.

alpha	1	2	3	4	5	10	25
(alpha * r) as % Surplus	0.02	0.04	0.06	0.08	0.1	0.2	0.5
Half life (in years)	35	17	12	9	7	3	1

[1] In a stationary scheme wherein the inflow and outflow of members guarantee a stable composition of members as to age and sex, the contribution rate each year is the same whatever the duration of the horizon. In case of an ageing scheme, the long run method results in a higher contribution rate than the one year horizon, because the former takes the future ageing into account when calculating the level contribution rate.

Figure 2



Deal 3: Collective Defined Contribution scheme

Deal 3 effectively is the counterpart of deal 1. Deal 3 is a collective Defined Contribution scheme, wherein the contribution rate P_t equals the level contribution rate P_t^*, and wherein the risk in the funding process is borne by all members with accrued benefits by a suitable adjustment in the offered indexation. The total indexation rate i_t is equal to the promised indexation which is linked to the actual inflation rate in t, π_t , plus the additional indexation rate i_t^{add} needed to close the balance of the fund at the end of the period:

$$P_t = P_t^* \tag{24}$$

$$i_t = \pi_t + i_t^{add} \tag{25}$$

The additional indexation rate i_t^{add} can be solved from the expression below:

$$i_t^{add} ABO_t = S_t \tag{26}$$

Note the additional indexation rate applies only to the accrued benefits as in this deal it is not allowed to shift any funding residual to the future.

We can rewrite the above expression for a stationary scheme into a more general form. The first step is to recognize the path-dependency of the term ABO. The actual value of ABO is related to the indexation given in the past. This actual value may deviate from the target value of ABO^* being the value of ABO when indexation in the past always has been given.

Let us rewrite the above expression into:

$$i_t^{add} ABO_t^* \gamma_t = S_t \tag{27}$$

with

$$\gamma_t = \frac{ABO_t}{ABO_t^*} \tag{28}$$

For a stationary scheme the expression $i_t^{add} ABO_t^* \gamma_t = S_t$ can now be rewritten as:[2]

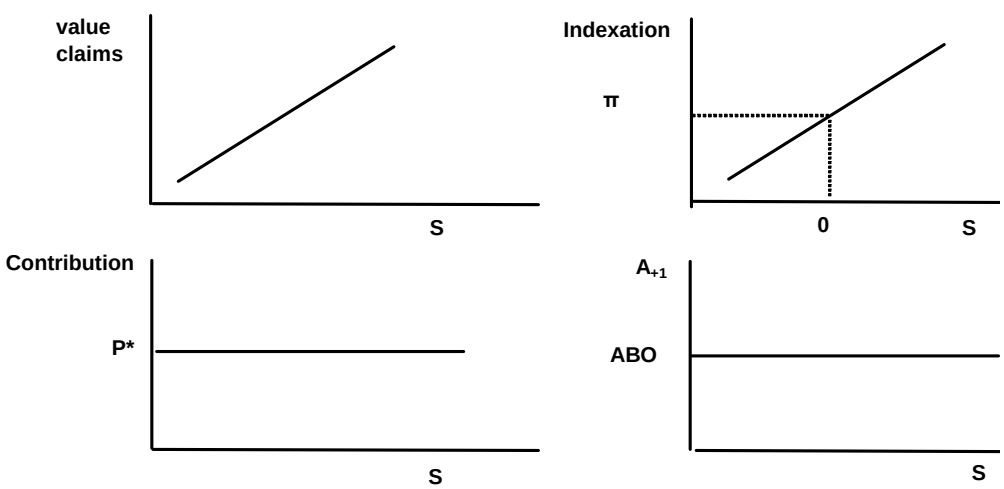
$$i_t^{add} = \frac{S_t}{\Pi_t} \frac{1}{\gamma_t} \frac{r}{b^* - p^*} \tag{29}$$

As is the case in deal 1, next period assets equals the present value of accrued benefits at the end of period t:

$$A_{t+1} = ABO_t \tag{30}$$

Figure 3 shows the graphical representations of the risk-bearing process in this plan:

Figure 3: Collective Defined Contribution Plan



[2] This result follows from the rewriting of expression (27) as follows: $i_t^{add} = \frac{S_t}{ABO_t^*} \frac{1}{\gamma_t} = \frac{S_t}{PBO_t^* - nBO_t^*} \frac{1}{\gamma_t} = \frac{S_t}{PBO_t^* - PVP_t^*} \frac{1}{\gamma_t} = \frac{S_t}{\frac{B}{r} - \frac{P}{r}} \frac{1}{\gamma_t} = \frac{rS_t}{\Pi b^* - \Pi p^*} \frac{1}{\gamma_t} = \frac{S_t}{\Pi_t} \frac{1}{\gamma_t} \frac{r}{b^* - p^*}$.

Deal 4: Collective Defined Contribution scheme with risk shifting over time

The volatility in the yearly indexation rate in Deal 3 may be very high because of the absorption of the shocks hitting the pension fund every period. If we allow for spreading risk over time we may adjust the expression for the additional indexation rate as follows:

$$i_t^{add} PBO_t = S_t \tag{31}$$

The capacity of risk-bearing is extended substantially by switching from accrued benefits to projected benefits.

The expression above states that a funding residu is spread out as either an equal cut or equal bonus to the indexation this year and all years during the horizon under consideration.

The above expression can be rewritten for a stationary scheme by making use of $PBO_t = B/r$ as follows:

$$i_t^{add} = \frac{S_t}{\Pi_t} \frac{1}{\delta_t} \frac{r}{b^*} \tag{32}$$

with

$$\delta_t = \frac{PBO_t}{PBO_t^*} \tag{33}$$

This result follows from the rewriting of expression $i_t^{add} PBO_t = S_t$ as follows:

$i_t^{add} = \frac{S_t}{PBO_t^*} \frac{1}{\delta_t} = \frac{S_t}{PBO_t^*} = \frac{S_t}{\frac{B}{r} - \frac{P}{r}} \frac{1}{\delta_t} = \frac{rS_t}{\Pi b^*} \frac{1}{\delta_t} = \frac{1}{\delta_t} \frac{S_t}{\Pi_t} \frac{r}{b^*}$

Note the similarity between Deal 4 and Deal 2. In Deal 4 it is the additional indexation that exactly matches the interest load of the funding residue rS_t whereas in Deal 2 it is the additional contribution which absorbs the interest load of the residu.

A generalization can also be applied to expression (32), where the factor β lies between $\frac{1}{\delta_t b^*}$ and $\frac{1}{\gamma_t (b^* - p^*)}$ and where Deal 3 and Deal 4 are included as two extreme cases[3]:

$$i_t^{add} = \beta \frac{rS_t}{\Pi_t} \qquad \frac{1}{\delta_t} \frac{1}{b^*} \leq \beta \leq \frac{1}{\gamma_t} \frac{1}{b^* - p^*} \tag{34}$$

[3] Note that $\frac{1}{\delta_t} \frac{1}{b^*} \frac{1}{\gamma_t} \frac{1}{b^* - p^*}$ as $b^* > (b^* - p^*)$ and $\delta_t > \lambda_t$, the latter because $\delta_t = \frac{PBO}{PBO^*} = \frac{ABO + newBO^*}{ABO* + newBO^*} > \lambda_t = \frac{ABO}{ABO^*}$

The dynamics in the surplus resp. in the wealth of the pension fund has to be adjusted accordingly as follows, where the value of the accrued benefits and the term value of the residu both will change due to the additional indexation:

$$\begin{aligned} A_{t+1} &= \underbrace{\left(1+\pi_t+i_t^{add}\right) ABO_{t-1}}_{+\,ABO_t} + \underbrace{\left(S_t - i_t^{add} ABO_{t-1}\right)}_{newS_t} \\ &= \qquad + ABO_t \qquad + \qquad newS_t \end{aligned} \tag{35}$$

So far, we have discussed four distinct pension deals, where each of them can be seen as a corner in the absorption of risks in the funding process. These variants also are corners in the allocation of risk to the involved generations, hence to intergenerational transfers of value and intergenerational risk-sharing. In between the corners, there are an infinite number of alternatives.

Now we turn to two hybrid variants as these variants are composed of components of a pure DB scheme and a collective scheme.

Deal 5:

Figure 4 below shows the main characteristics of this plan. In deal 4 risk is allocated to current and future indexation. The indexation may even be negative. Suppose the participants strife to at least a 0% guarantee as to their benefits. They buy this downward protection by selling the upward part of the indexation risk. It is assumed here that the maximum indexation is equal to two times the actual inflation rate. When the funding residu falls below the lower bound or exceeds the upper bound, then current workers absorb this part of the residu.

The risk allocation is according to the set of rules below:

$$\begin{aligned} [1] \quad \text{when } \frac{S_t}{ABO_t^*} &< -\pi_t \text{ then} \\ i^{add} &= -\pi_t \\ P^{add} &= -S_t - \pi_t ABO_t^* \end{aligned}$$

$$
\begin{aligned}
[2] \quad \text{when} -\pi_t &\leq \frac{S_t}{ABO_t^*} \leq \pi_t \text{ then} \\
i^{add} &= \frac{S_t}{ABO_t^*} \\
P^{add} &= 0
\end{aligned}
$$

$$
\begin{aligned}
[3] \quad \text{when } \frac{S_t}{ABO_t^*} &> \pi_t \text{ then} \\
i^{add} &= \pi_t \\
P^{add} &= -S_t + \pi_t ABO_t^*
\end{aligned}
$$

Figure 4: Deal 5

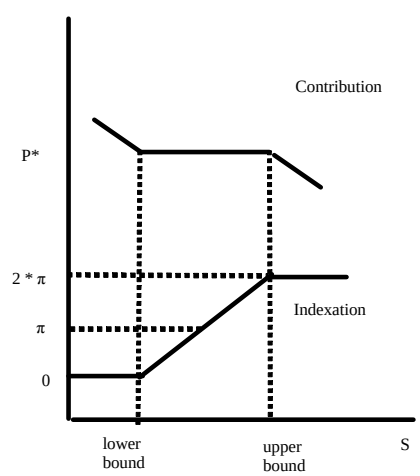
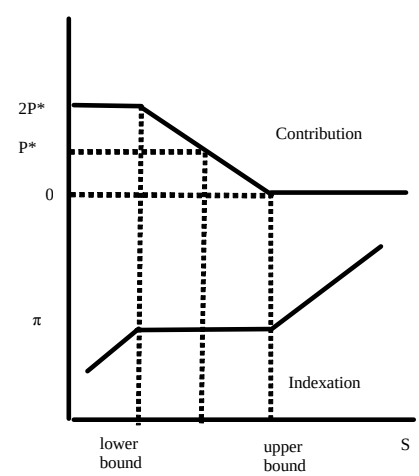


Figure 5: Deal 6



2.3.2 Deal 6:

Figure 5 shows the main characteristics of this plan. In deal 2 risk is allocated to current and future contributions. Suppose the workers accept a maximum contribution equal to $2P^*$. They buy this protection by giving up the possibility that in good times the contribution rate may fall below zero.

When the funding residu falls below the lower bound or exceeds the upper bound, then current participants absorb this part of the residu by accepting indexation risk.

The risk allocation is according to the set of rules below:

$$\begin{aligned} [1] \quad \text{when } S_t &< -P^* \text{ then} \\ i^{add} &= \frac{S_t + P^*}{ABO_t} \\ P^{add} &= P^* \end{aligned}$$

$$\begin{aligned} [2] \quad \text{when} - P^* &\leq S_t \leq P^* \text{ then} \\ i^{add} &= 0 \\ P^{add} &= -S_t \end{aligned}$$

$$\begin{aligned} [3] \quad \text{when} S_t &> P^* \text{ then} \\ i^{add} &= \frac{S_t - P^*}{ABO_t} \\ P^{add} &= -P^* \end{aligned}$$

3 Analysis of the pension deals

We now turn to an analysis of the various pension deals. We look at various aspects of the deals. First, we calculate the market value of the pension deal from the point of view of an individual member. We then turn to an analysis using utility functions, which is more appropriate for non-marketable pension fund membership.

3.1 Economic and Financial Risk factors

In this section we describe the economic environment of the pension fund, and model the returns on investmet assets and the evolution of the liabilities. In the previous section we derived the expression for the dynamics in the (nominal) surplus:

$$dS_t = [(P_t - P_t^*) - (B_t^* - B_t) + R_t A_t - (r + \pi_t)(PVB_t - PVP_t)]dt \quad (36)$$

The sources of risk in equation are the random asset returns, inflation, and the real interest rate. With this structure, we can easily simulate the actual benefit stream, X_s, into the future for any $s > t$. To simulate the model, we need to make assumptions about the probability distribution of asset returns and inflation.

We adopt the structure of the economy in Brennan and Xia (2002). Their model has four underlying risk factors: stock returns, expected inflation, unexpected inflation, and real interest rate shocks. For simplicity, we assume for now that real interest rates are non-stochastic. Nominal asset values are given by

$$dA_t/A_t = \mu dt + \sigma dZ_t \tag{37}$$

Hence, asset values follow a geometric Brownian motion with drift. Inflation is modeled as the sum of expected inflation π_t^e and an independent unexpected inflation shock. Expected inflation follows a mean reverting process

$$d\pi_t^e = a(\bar{\pi} - \pi_t^e)dt + \sigma_\pi dZ_{\pi,t} \tag{38}$$

The price level changes according to

$$d\Pi_t/\Pi_t = \pi_t^e dt + \sigma_u dZ_{u,t} \tag{39}$$

where the last term is the unexpected inflation.

The model also gives an easy way to determine the market value of financial claims. One such financial claim is the cash flows (contributions and benefits) generated by membership of a pension fund. The deflator for real claims in this setting is determined by

$$dD_t^{real}/D_t^{real} = -rdt - \lambda_t dZ_t - \lambda_\pi dZ_{\pi,t} - \lambda_u dZ_{u,t} \tag{40}$$

where the λ's are market prices of risk. Notice that to prevent arbitrage, the market price of risk for the asset returns must be time-varying when μ is fixed

$$\lambda_t = \frac{\mu - r - \pi_t^e}{\sigma} \tag{41}$$

For nominal claims, the deflator is $D_t = D_t^{real}/\Pi_t$ which evolves as

$$dD_t/D_t = -(r + \pi_t^e)dt - \lambda_t dZ_t - \lambda_\pi dZ_{\pi,t} - (\lambda_u - \sigma_u)dZ_{u,t} \tag{42}$$

Table 1 shows the default values for the model parameters to be used in the calculations. The parameters are partly based on estimates reported in

Brennan and Xia (2002), but some values are superseded to numbers of our own preference. For example, we set the average inflation at 2%, in line with the informal ECB target for the euro area. We also assume the real interest rate is constant and equals 2%. Finally, the expected return on equity is assumed to be 8%, implying an equity premium of around 4%, which is in line with the long run estimates in Fama and French (2002).

3.2 Assumptions on the pension fund

We assume a going-concern pension fund with stationary member profiles. In principle, each age cohort finances his/her own pension. The pension fund has an average pay system. Besides this pension income, we assume there is a state pension of 10000 (euro, in real terms) available after retirement. We assume that the wage inflation is identical to the price inflation. An average career maker has a linear real salary path which increases linearly from 20000 to 39500 during his career (from age 25 to 64). The pension buildup percentage is assumed to be 2.25% per year over the pensionable salary, this is salary minus the state pension income. The real pension income b is determined by $\sum_{t=25}^{64} (sal_t - statepension_t) \cdot 2.25\%$ =17,775 (euro, in real terms). The real cost price contribution p is solved from the following equivalent:

$$\sum_{t=0}^{64-25} p(1+r)^{-t} = \sum_{t=65-25}^{79-25} b(1+r)^{-t}$$

This gives p = 3,759 euro, if assuming $r = 2\%$.

We distinguish two financial market settings to which the pension fund have access. In section 3.3.1, the pension fund invests the assets into stocks and Index Linked Bonds (which realizes the risk-free return in the portfolio). In section 3.3.2, we will replace the ILB with nominal bonds. All the calculations are based on collective account setting, where the investments of all cohorts are pooled into one account. The surplus of that account will trigger the conditional contribution and indexation rules applied in the pension deals. Within the collective setting, we also run a generational accounting, which tracks the asset development of an individual or cohort within the collective account. The individual accounts are only synpothetic accounts, however, because the contribution and indexation rules are defined by the collective effects.

3.3 Valuation of the pension deals

The first step in the analysis is a valuation exercise. We look at all the cash flows generated by the pension deals and value these using the deflator method. This assumes all the risks are tradable. For an individual member this may be difficult once he stepped into a pension contract, but a market valuation is useful ex-ante, when deciding which pension offer to accept, or when transferring pensions from one fund to another. From the point of view of the fund member, the value of the pension deal is the value of the actual benefits minus the premiums. For a member with retirement date T, this can be modeled as

$$PV = \mathsf{E}\left[-\int_0^T M_t P_t dt + \int_T^\infty M_t B_t dt\right] \tag{43}$$

where P_t and B_t are the actual premiums received and benefits paid in year t. The discount factor M_t is the product of the deflator for risky cash flows and the survivor function (probability of being alive at time t).Lancaster (1990, p.9) defines the survivor function in terms of the mortality rate θ_t. He shows that the survivor function equals $\exp\left\{-\int_0^t \theta_s ds\right\}$. It follows that M_t evolves according to

$$dM_t/M_t = -(r + \pi^e + \theta_t)dt - \lambda_t dZ_t - \lambda_\pi dZ_{\pi,t} - (\lambda_u - \sigma_u)dZ_{u,t} \tag{44}$$

In the numerical examples below, the assumptions on the mortality are quite simple. Working life starts at 25 and lasts until 65, when there is mandatory retirement. To simplify the calculations, we assume all agents die at 80.

3.3.1 Valuation of the pension deals with ILB and equity

Table 2 shows the valuation of the six pension deals outlined before, with an initial funding ratio of 100%. For each deal, we assume three different asset mixes: 100% Index Linked Bonds; 50% ILB and 50% equity; and 100% equity. The table shows a number of interesting characteristics of the pension deals. First, the table provides an estimate of the present value of the actual contributions made to the fund over the lifetime; in several of these deals the actual contributions may deviate from the cost-price contribution. The standard deviations of each present value estimates, which are reported next to the corresponding present values, provide important messages about the uncertainties associated with each pension deals and asset mixes.[4] Column

[4] The differences in the standard deviations are due to 1) the time effect (i.e. the further away into the time horizon, the larger the std. dev.) and 2) the riskines of the asset mix.

(2) provides the present value of the additional contributions. Notice that these numbers may be negative when the value of the actual contributions (column 3) is below the cost price contribution (column 1). The table further provides the present value of the actual benefits in column (4) and the value of the remaining assets in column (5). These remaining assest are what's left in the individual accounts when the person dies. This number may be negative. i.e. individual members may leave a deficit, which has to be absorbed by the pension fund. The total transfer (column 6) adds up the value of contributions and the remaining assets, and subtracts the value of the benefits recieved. This is exactly the value of the net transfers that the individual member makes to the fund. This number can also be seen as minus the net present value (for the individual) of joining the pension deal. In most of the cases, the dispersion of the total transfer (column *std (6)*) is very large, which indicating that substantial amount of transfers. Table 3 contains the same data, but expressed in terms of the initial gross wage, which is easier to read and to interpret. The quantile plots of several interested quantities (Figure 6 to18) are based on the 50% ILB and 50% equity assumption.

For deals 1 and 2, the present value (and also the standard deviation) of the pension deal is exactly zero for the 100% index linked bond investment case. This is no surpirse, as with this investment strategy the cost price contribution is exactly the right price to guarantee the benefits. Also, since the growth of the assets exactly tracks the liabilities (inflation), there is no real risk and no remaining assets in this deal. With 50% or 100% equity, the picture changes. First, let's take a look at the size of the additional contributions. The standard deviation of the additional contributions are quite high (nearly twice or four times as large as the cost price contributions). To obtain more insights, Figure 6 plots the quantiles of the additonal contributions. Two things are clear from the figure. First, the additonal contributions are more often negative than positive, and the distribution is skewed. This is due to the equity premium: due to the high average return on equity, contribution reductions are more frequent (and also larger) than contribution increases. However, this does not add value to the pension deal: the lower contributions occur in scenarios where the equity returns are high, but the deflators in such scenarios are low. The net effect on the value of the deal is very small indeed. For deal 2, where the additional contributions are spread out over time, the same intuition holds. The main difference with deal 1 is the smaller dispersion of the additional contributions, but very large dispersion of the remaining asset, which indicating substantial value

transfers taking place at the end of the life. Figure 14 shows an additonal interesting feature of deal 2. Because additonal contributions are spread out over time, and because the additional contributions are most often negative, often large surplusses are left at the end of each year. Although the dispersion of the remaining asset is large, the market value of the remaining assets in individual account is low, because positive surpluses occur only when equity returns are high and the delfator takes low values.

The results for deal 3 and 4 (where asset returns are absorbed by adjusting the benefits) the story is pretty much the same. In present value terms, the deviations from the no-risk setting (i.e. 100% ILB) are small. However, the dispersion of the actual benefits (Figure 10) and surplusses at the end of each year (Figure 15) are still quite large. The dispersion of the total transfers is significantly reduced (column *std (6)*) comparing with that of deal 1 and 2. Another point to make is, using ABO as indexation base (as in deal 3) or using the PBO as indexation base (as in deal 4) does not affect the actual benefit levels. But it does affect the volativity of the additional indexations (Figure 17). With risk-sharing, deal 4 provides less volatile additional indexations. By adjusing indexations in the way the we specified, the present value of the acutal benefits is higher than the actual contributions, especially in case of 100% ILB, This results in value transfers without stoking additional contributions or negative remaining asset. Because of this, in Figure 17, the quantiles of the addtional indextion take a slightly downturn trend. This may have impact on the sustainability of the pension fund in the long-run.

The analysis of deal 5 and 6 is more complex. Recall that in deal 5, the additonal benefits are adjusted up to limits, and the additional contributions pick up the extreme (high and low) returns. In deal 6 it is the other way around: contributions can be adjusted up to a limit, and the benefits pick up the extremes. Perhaps not surprisingly, the dispersion of additional contributions in deal 5 is still large, and comparable to deal 1 (see Figure 7). In present value terms, the limits on indexation and contributions now have a substantial impact. For deal 5, more equity in the portfolio leads to lower contributions and also to a lower present value of the contributions. However, the value of the actual benefits (that pick up the bad returns!) is also lower. In addition, sometimes large deficits are left to the fund. The net effect of these three terms is relatively small, however. On the other hand, for more ILB in the portfolio, since the indexation is limited within a non-negative range, the actual benefits are pushed up even higher than

that of deal 3 or 4. The net present value of the pension deal 5 is not very different from the value of the other deals. In many ways, deal 6 is the mirror image of deal 5: additonal contributions are higher with more equity, but the value of the benefits is correspondingly higher too. Again, the net present value of the deal is relatively small.

3.3.2 Valuation of the pension deals with Nominal Bonds and equity

In a market setting where ILB is not available, investing in nominal bonds is an obvious alternative. Here we assume the duration of the nominal bond portfolio is 6. Inflation risks are not perfectly hedged by this portfolio. But, replacing ILB with nominal bonds in the asset mix does not change the overall picture of the above stories.

For deal 1 and 2, although the additional contribuitions are more skewed to the negative side (Figure 19), the present value of the additional contribution is very small. This is because the risk premium of the bonds is corrected by the deflators. For deal 2, the surpluses of the pension fund are accumulated up on average (Figure 23). Furthermore, the dispersions of the surpluses are larger for the nominal bonds strategy. The intuition is the same, because additonal contributions are spread out over time, and because the additional contributions are most often negative, often large surplusses are left at the end of each year. The total transfers are substantial (Table 6 and 7, column *std (6)*), although they are very small in present value terms.

For deal 3 and 4, in Figure 21 we observe that, the quantiles of the actual benefits are higher than the case with ILB strategy, but the present value of the actual benefit is lower. Once again, the risk premium in the nominal bond is corrected by the deflators. Comparing the standard deviation of the total transfers with that of deal 1 and 2, we see that, by adjusting indexations instead of contributions, deal 3 and 4 reduce the dispersion of value transfers significantly (Table 6 and 7, column *std (6)*). Table 6 and7 also show that, because of the asset diversification, the mixed portfolio (50% bonds and 50% equity) achieves a smaller dispersion in the transfers.

Deal 5 and 6 are combinations of the previous four deals, and thus the intuitions from the previous results hold. For deal 5 and 6, the large reductions on the addtional contribution lead to larger reductions on the actual benefits, comparing with the ILB strategy. The dispersions of the surpluses and remaining assets are very large.

3.4 Utility analysis

This section compares the pension deals in utility terms, with special attention to the order of preferences regarding the different deals and different asset mix from a newly-entry-cohort's point of view.

The setup of the analysis is the following: The consumption in every period equals labor income minus contributions before retirement, and benefits plus a fixed state pension after retirement. This assumes that the only savings of the consumer are in the pension fund! Utility is the expectation of the discounted sum of the utility of consumption in every period.[5] To accommodate the possible negative consumptions occuring in deal 1 and 5, we assume that the pension fund member has a constant absolute risk aversion (CARA) utility function with absolute risk aversion parameter A.[6] Since people prefer smoother consumption pattern, for a given average level of consumption, the larger the volatility of the consumption stream, the lower the utility is. Formally, the utility function takes the form:

$$E_{t_0}[U(c_0, c_1, ..., c_{N-1})] = E_{t_0}\left[\sum r^{-t} u(c_t)\right]$$

with

$$u(c_t) = -\exp(-A * c_t)$$

Results in Table 5 and Table 9 show many things in common: deal 1 and 5 have very large contribution fluctuations, therefore sometimes extremely low consumption and the consumer prefers a very safe investment strategy. For all the other deals, the consumer prefers a high equity stake (how much depends on his risk aversion). This is the effect of the high equity premium: although the equity premium doesn't add anyting in market value terms, in utility terms, equity investments seem to be a good deal.

From utility terms, asset mix with nominal bonds provides slightly higher utlities than portflio with ILB. This result is due to two effects: notice that

[5] In this paper, lifetime consumption streams are considered when determinating the utilities. This is different from taking the expected terminal utility as objective function in DC plan evaluations.

[6] When consumption streams remain positive in all the scenarios, CRRA utility function becomes applicable, e.g. for Deal 2, 3, 4 and 6. CRRA utility takes the form:

$$u(c_t) = c_t^{(1-\gamma)}/(1-\gamma)$$

The results in Table 5 and 7 show that the order of preferences are consistent with the two types of utility functions.

with nominal bonds in the portfolio, the fluctuations in the consumption stream is increased, hence the utility will be lower; on the other hand, the average level of the consumption stream is increased due to the risk premium in the nominal bonds, hence the utility will be higher. The second effect dominates, given our current set of model parameters.

4 Conclusion

In this paper, we have presented a formal framework to model the transfers of value between generations in a pension fund based on intergenerational risk-sharing. We have made use of the institutional setting of public sector pension funds or industry pension funds based on intergenerational risksharing, where pension benefits and contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). Explicit risk allocation rules have been defined, specifying who of the stakeholders, when, and to what extent is taking part in risk-bearing. Using contingent claims valuation methods (deflators), we have calculated these transfers and the distribution of value across generations for alternative sets of risk-allocation. The alternative pension deals have no impact on the total economic value of the combined stakes of the stakeholders, however they will imply different distribution of the value amongst the stakeholders. Apart from risk allocation rules, choices with respect to the asset mix also will have impact on the value distribution. An investment strategy with 100% index linked bond guarantee a full match between the assets and the value of indexed liabilities. No transfers of value between generations will occur. Any other asset allocation will lead to substantial transfers, where the risk allocation rules determine who, when and to what extent is bearing the risk.

Utility analysis is added to the framework with the aim to order the alternative settings as to risk-allocation and the asset mix. The representative individu makes a trade-off between the level and the degree of riskiness of the consumption stream over the life time. The contribution instruments have a large impact on the volatility of consumption streams, especially in deal 1 and 5, where even negative consumptions are possible. The volatility of additional indexation per period does not affect the actual benefit levels significantly, i.e. the volatility in additional indexation is smoothed out in the value of accrued liabilities.

The results suggest that individuals generally prefer risk-taking above a riskless position in index-linked bonds. It turns out that the increase in

utility because of higher consumption level over the lifecycle more than outweigh the decrease in utility because of extra risk. The consumer prefers a high equity stake (how much depends on his risk aversion): although the equity premium doesn't add anyting in market value terms, in utility terms, equity investments seem to be a good deal. This conclusion prevails as long as funding risks can be spread out over time, in particular to absorb a funding deficit or funding surplus by adjusting the indexation of accrued liabilities and/or by adjusting the contribution rate during a long horizon. The conclusion does not hold when all the funding risk has to be absorbed within one period by an appropriate adjustment in the contribution rate. For such a deal, the individual prefers a riskless position in order to avoid the huge shocks in the year to year consumtion level during the active period.

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Table 1: Default parameters for the stochastic models

parameter	value
μ	0.08
r	0.02
π	0.02
a	0.2
σ	0.20
σ_π	0.01
σ_u	0.01
λ_π	-0.2
λ_u	-0.2
$\rho_{S\pi}$	-0.05

Table 2: Present Value of pension deals (in euros) (Index-Linked Bonds and Equity assumption)

	cost price		add Contrib		act. Contrib		act. Benefit		rem. Asset		transfer	
	(1)	*std (1)*	(2)	*std (2)*	(3)	*std (3)*	(4)	*std (4)*	(5)	*std (5)*	(6)	*std (6)*
D1	104935	*199408*	0	*0*	104935	*199408*	104935	*837018*	0	*0*	0	*0*
	104935	*199408*	23	*197279*	104959	*325800*	104935	*837018*	-2191	*439535*	-2167	*467414*
	104935	*199408*	47	*394559*	104982	*500365*	104935	*837018*	-2992	*792024*	-2945	*837269*
D2	104935	*199408*	0	*0*	104935	*199408*	104935	*837018*	0	*0*	0	*0*
	104935	*199408*	314	*114791*	105249	*285725*	104935	*837018*	-2099	*578766*	-1785	*582339*
	104935	*199408*	377	*207042*	105312	*363437*	104935	*837018*	-1833	*898027*	-1456	*910017*
D3	104935	*199408*	0	*0*	104935	*199408*	106999	*910880*	0	*0*	-2064	*78303*
	104935	*199408*	0	*0*	104935	*199408*	105363	*668803*	-230	*103825*	-658	*269240*
	104935	*199408*	0	*0*	104935	*199408*	104765	*619064*	43	*163228*	214	*482128*
D4	104935	*199408*	0	*0*	104935	*199408*	106999	*910880*	0	*0*	-2064	*78303*
	104935	*199408*	0	*0*	104935	*199408*	105432	*666312*	-370	*97403*	-867	*284025*
	104935	*199408*	0	*0*	104935	*199408*	104984	*630222*	-181	*157428*	-230	*516626*
D5	104935	*199408*	1470	*3010*	106406	*201816*	108028	*913541*	394	*10299*	-1229	*79904*
	104935	*199408*	-5321	*88611*	99614	*270297*	96091	*708802*	-3000	*272076*	523	*322499*
	104935	*199408*	-5818	*214524*	99118	*371027*	94836	*689509*	-3237	*578377*	1045	*611533*
D6	104935	*199408*	0	*0*	104935	*199408*	106999	*910880*	0	*0*	-2064	*78303*
	104935	*199408*	6784	*43009*	111719	*233815*	110295	*726405*	1346	*153390*	2771	*260037*
	104935	*199408*	4632	*41077*	109567	*232269*	107422	*635428*	1754	*200240*	3899	*468568*

Table 3: Present Value of pension deals relative to the start gross salary of 20000 euro (Index-Linked Bonds and Equity assumption)

	cost price		add Contrib		act. Contrib		act. Benefit		rem. Asset		transfer	
	(1)	*std (1)*	(2)	*std (2)*	(3)	*std (3)*	(4)	*std (4)*	(5)	*std (5)*	(6)	*std (6)*
D1	5.25	*10*	0	*0*	5.25	*10*	5.25	*42*	0	*0*	0	*0*
	5.25	*10*	0	*10*	5.25	*16*	5.25	*42*	-0.11	*22*	-0.11	*23*
	5.25	*10*	0.01	*20*	5.25	*25*	5.25	*42*	-0.15	*40*	-0.15	*42*
D2	5.25	*10*	0	*0*	5.25	*10*	5.25	*42*	0	*0*	0	*0*
	5.25	*10*	0.02	*6*	5.26	*14*	5.25	*42*	-0.1	*29*	-0.09	*29*
	5.25	*10*	0.02	*10*	5.27	*18*	5.25	*42*	-0.09	*45*	-0.07	*46*
D3	5.25	*10*	0.0	*0*	5.25	*10*	5.35	*46*	0	*0*	-0.1	*4*
	5.25	*10*	0.0	*0*	5.25	*10*	5.27	*33*	-0.01	*5*	-0.03	*13*
	5.25	*10*	0.0	*0*	5.25	*10*	5.24	*31*	0	*8*	0.01	*24*
D4	5.25	*10*	0.0	*0*	5.25	*10*	5.35	*46*	0	*0*	-0.1	*4*
	5.25	*10*	0.0	*0*	5.25	*10*	5.27	*33*	-0.02	*5*	-0.04	*14*
	5.25	*10*	0.0	*0*	5.25	*10*	5.25	*32*	-0.01	*8*	-0.01	*26*
D5	5.25	*10*	0.07	*0*	5.32	*10*	5.40	*46*	0.02	*1*	-0.06	*4*
	5.25	*10*	-0.27	*4*	4.98	*14*	4.80	*35*	-0.15	*14*	0.03	*16*
	5.25	*10*	-0.29	*11*	4.96	*19*	4.74	*34*	-0.16	*29*	0.05	*31*
D6	5.25	*10*	0	*0*	5.25	*10*	5.35	*46*	0	*0*	-0.1	*4*
	5.25	*10*	0.34	*2*	5.59	*12*	5.51	*36*	0.07	*8*	0.14	*13*
	5.25	*10*	0.23	*2*	5.48	*12*	5.37	*32*	0.09	*10*	0.19	*23*

Table 4: Total Transfers (Index-Linked Bonds and Equity assumption)

	equity %	D1	D2	D3	D4	D5	D6
absolute	0%	0	0	-2064	-2064	-1229	-2064
amount	50%	-2167	-1785	-658	-867	523	2771
	100%	-2945	-1456	214	-230	1045	3899
relative	0%	0	0	-0.1	-0.1	-0.06	-0.1
to salary	50%	-0.11	-0.09	-0.03	-0.04	0.03	0.14
	100%	-0.15	-0.07	0.01	-0.01	0.05	0.19

Table 5: Utility Values (Index-Linked Bonds and Equity assumption)

	Equity as %	CARA utility A = [2 6 8]			CRRA utility gamma = [2 3 5]		
D1	0%	-20.4617	**-7.8020**	**-4.8959**	N.A.		
	50%	**-20.2190**	-9.0977	-6.7515			
	100%	-21.3219	-17.4345	-21.4769			
D2	0%	-20.4617	-7.8020	-4.8959	-0.1401	-0.3058	-3.3565
	50%	-19.8626	-7.2825	-4.5140	-0.1340	-0.2846	**-3.0658**
	100%	**-19.2193**	**-6.9426**	**-4.3104**	**-0.1299**	**-0.2753**	-3.0739
D3	0%	-20.4617	-7.8020	-4.8959	-0.1401	-0.3058	-3.3565
	50%	-19.7233	-7.3420	**-4.6024**	-0.1346	**-0.2912**	**-3.2442**
	100%	**-19.3066**	**-7.3090**	-4.6197	**-0.1334**	-0.2925	-3.3490
D4	0%	-20.4617	-7.8020	-4.8959	-0.1401	-0.3058	-3.3565
	50%	-19.7102	-7.3378	**-4.6003**	-0.1346	**-0.2911**	**-3.2437**
	100%	**-19.3027**	**-7.3109**	-4.6212	**-0.1334**	-0.2926	-3.3482
D5	0%	-20.4808	**-7.8359**	**-4.9292**	N.A.		
	50%	**-20.0608**	-8.5061	-6.0172			
	100%	-20.9723	-15.5546	-18.2703			
D6	0%	-20.4617	-7.8020	-4.8959	-0.1401	**-0.3058**	**-3.3565**
	50%	-19.7125	-7.3925	**-4.6799**	-0.1364	-0.3064	-4.0441
	100%	**-19.2608**	**-7.3393**	-4.6857	**-0.1349**	-0.3077	-4.1806

Table 6: PV in euros (Nominal Bonds and Equity assumption)

	cost price		add Contrib		act. Contrib		act. Benefit		rem. Asset		transfer	
	(1)	*std (1)*	(2)	*std (2)*	(3)	*std (3)*	(4)	*std (4)*	(5)	*std (5)*	(6)	*std (6)*
D1	104935	*199408*	175	*63437*	105110	*245355*	104935	*837018*	565	*238651*	740	*261781*
	104935	*199408*	111	*205253*	105046	*342874*	104935	*837018*	-1920	*410166*	-1809	*451941*
	104935	*199408*	47	*394559*	104982	*500365*	104935	*837018*	-2992	*792024*	-2945	*837246*
D2	104935	*199408*	453	*57905*	105389	*247157*	104935	*837018*	140	*199909*	593	*216608*
	104935	*199408*	460	*121808*	105396	*300358*	104935	*837018*	-1929	*562528*	-1468	*566566*
	104935	*199408*	377	*207042*	105312	*363437*	104935	*837018*	-1833	*898027*	-1456	*910209*
D3	104935	*199408*	0	*0*	104935	*199408*	106681	*802413*	-62	*38841*	-1807	*82676*
	104935	*199408*	0	*0*	104935	*199408*	105381	*623277*	-225	*103382*	-670	*269759*
	104935	*199408*	0	*0*	104935	*199408*	104765	*619064*	43	*163228*	214	*482132*
D4	104935	*199408*	0	*0*	104935	*199408*	106649	*798961*	-2	*36689*	-1716	*84599*
	104935	*199408*	0	*0*	104935	*199408*	105435	*619167*	-333	*97484*	-832	*283252*
	104935	*199408*	0	*0*	104935	*199408*	104984	*630222*	-181	*157428*	-230	*516626*
D5	104935	*199408*	-25804	*40799*	79131	*232048*	87773	*710533*	-6610	*104907*	-15252	*140783*
	104935	*199408*	-22162	*91062*	82773	*272257*	91011	*659645*	-4096	*246057*	-12333	*318599*
	104935	*199408*	-19381	*216143*	85555	*372322*	94836	*689509*	-3237	*578377*	-12518	*609901*
D6	104935	*199408*	1277	*30679*	106212	*223951*	107668	*834939*	620	*112273*	-835	*132496*
	104935	*199408*	6165	*41814*	111100	*232860*	109802	*681299*	1301	*142362*	2600	*251508*
	104935	*199408*	4632	*41077*	109567	*232269*	107422	*635428*	1754	*200240*	3899	*468564*

Table 7: PV relative to the start gross salary of 20000 euro. (Nominal Bonds and Equity assumption)

	cost price		add Contrib		act. Contrib		act. Benefit		rem. Asset		transfer	
	(1)	*std (1)*	(2)	*std (2)*	(3)	*std (3)*	(4)	*std (4)*	(5)	*std (5)*	(6)	*std (6)*
D1	5.25	*10*	0.01	*3*	5.26	*12*	5.25	*42*	0.03	*12*	0.04	*13*
	5.25	*10*	0.01	*10*	5.25	*17*	5.25	*42*	-0.1	*21*	-0.09	*23*
	5.25	*10*	0	*20*	5.25	*25*	5.25	*42*	-0.15	*40*	-0.15	*42*
D2	5.25	*10*	0.02	*3*	5.27	*12*	5.25	*42*	0.01	*10*	0.03	*11*
	5.25	*10*	0.02	*6*	5.27	*15*	5.25	*42*	-0.1	*28*	-0.07	*28*
	5.25	*10*	0.02	*10*	5.27	*18*	5.25	*42*	-0.09	*45*	-0.07	*46*
D3	5.25	*10*	0.0	*0*	5.25	*10*	5.33	*40*	0	*2*	-0.09	*4*
	5.25	*10*	0.0	*0*	5.25	*10*	5.27	*31*	-0.01	*5*	-0.03	*13*
	5.25	*10*	0.0	*0*	5.25	*10*	5.24	*31*	0	*8*	0.01	*24*
D4	5.25	*10*	0.0	*0*	5.25	*10*	5.33	*40*	0	*2*	-0.09	*4*
	5.25	*10*	0.0	*0*	5.25	*10*	5.27	*31*	-0.02	*5*	-0.04	*14*
	5.25	*10*	0.0	*0*	5.25	*10*	5.25	*32*	-0.01	*8*	-0.01	*26*
D5	5.25	*10*	-1.29	*2*	3.96	*12*	4.39	*36*	-0.33	*5*	-0.76	*7*
	5.25	*10*	-1.11	*5*	4.14	*14*	4.55	*33*	-0.2	*12*	-0.62	*16*
	5.25	*10*	-0.97	*11*	4.28	*19*	4.74	*34*	-0.16	*29*	-0.63	*30*
D6	5.25	*10*	0.06	*2*	5.31	*11*	5.38	*42*	0.03	*6*	-0.04	*7*
	5.25	*10*	0.31	*2*	5.56	*12*	5.49	*34*	0.07	*7*	0.13	*13*
	5.25	*10*	0.23	*2*	5.48	*12*	5.37	*32*	0.09	*10*	0.19	*23*

Table 8: Total Transfers (Nominal Bonds and Equity assumption)

	equity %	D1	D2	D3	D4	D5	D6
absolute	0%	740	593	-1807	-1716	-15252	-835
amount	50%	-1809	-1468	-670	-832	-12333	2600
	100%	-2945	-1456	214	-230	-12518	3899
relative	0%	0.04	0.03	-0.09	-0.09	-0.76	-0.04
to salary	50%	-0.09	-0.07	-0.03	-0.04	-0.62	0.13
	100%	-0.15	-0.07	0.01	-0.01	-0.63	0.19

Table 9: utility values (Nominal Bonds and Equity assumption)

	Equity as %	CARA utility A = [2 6 8]			CRRA utility gamma = [2 3 5]		
D1	0%	-20.1435	**-7.6810**	**-4.8967**	N.A.		
	50%	**-20.0620**	-9.0163	-6.7356			
	100%	-21.3219	-17.4345	-21.4769			
D2	0%	-20.2231	-7.5697	-4.7165	-137.3	-295.2	-3181.5
	50%	-19.6969	-7.1497	-4.4184	-132.4	-279.1	**-2988.0**
	100%	**-19.2193**	**-6.9426**	**-4.3104**	**-129.9**	**-275.3**	-3073.9
D3	0%	-20.1476	-7.5328	-4.7031	-137.0	-296.0	-3252.8
	50%	-19.5216	**-7.2435**	**-4.5426**	**-133.3**	**-287.9**	**-3216.5**
	100%	**-19.3066**	-7.3090	-4.6197	-133.4	-292.5	-3349.0
D4	0%	-20.1416	-7.5282	-4.7000	-137.0	-295.9	-3251.4
	50%	-19.5047	**-7.2388**	**-4.5404**	**-133.2**	**-287.8**	**-3215.9**
	100%	**-19.3027**	-7.3109	-4.6212	-133.4	-292.6	-3348.2
D5	0%	-20.0699	**-7.4633**	**-4.6495**	N.A.		
	50%	**-19.8835**	-8.3153	-5.8440			
	100%	-20.9723	-15.5546	-18.2703			
D6	0%	-20.1157	-7.5267	-4.7143	-137.6	-302.6	**-3679.3**
	50%	-19.4956	**-7.2635**	**-4.5927**	**-134.7**	**-301.0**	-3952.5
	100%	**-19.2608**	-7.3393	-4.6857	-134.9	-307.7	-4180.6

Figure 6: 20th, 50th and 80th quantile of the additional contribution for Deal 1 (thin curve) and Deal 2 (curve with stars) (50% Index-Linked Bonds and 50% Equity assumption)

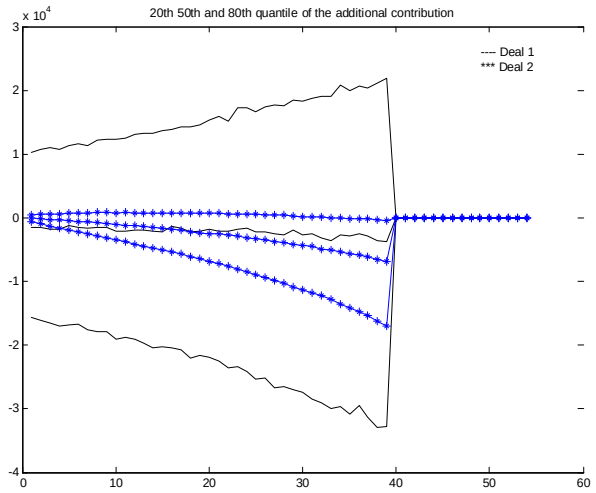

Figure 7: 20th, 50th and 80th quantile of the additional contribution for Deal 5 (thin curve) and Deal 6 (curve with stars) (50% Index-Linked Bonds and 50% Equity assumption)

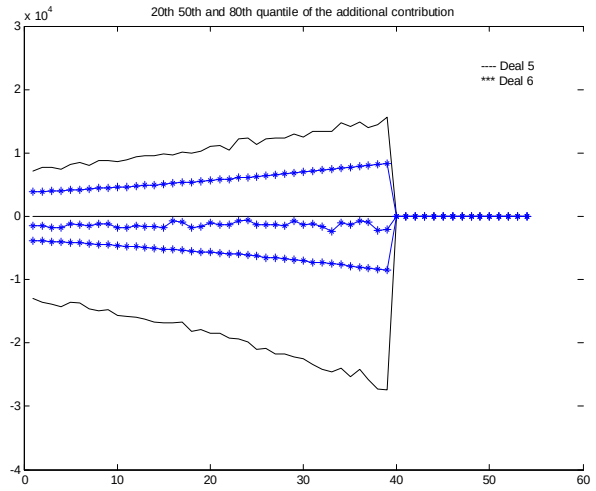

Figure 8: 20th, 50th and 80th quantile of the additional contribution for Deal 1 (thin curve) and Deal 2 (curve with stars), relative to the starting gross salary of 20000 euro (indexed) (50% Index-Linked Bonds and 50% Equity assumption)

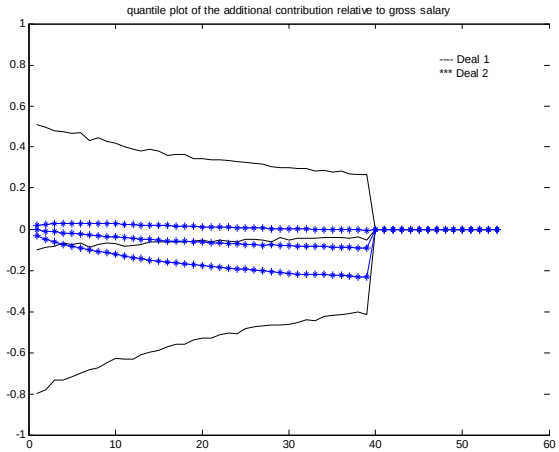

Figure 9: 20th, 50th and 80th quantile of the additional contribution for Deal 5 (thin curve) and Deal 6 (curve with stars), relative to the starting gross salary of 20000 euro (indexed) (50% Index-Linked Bonds and 50% Equity assumption)

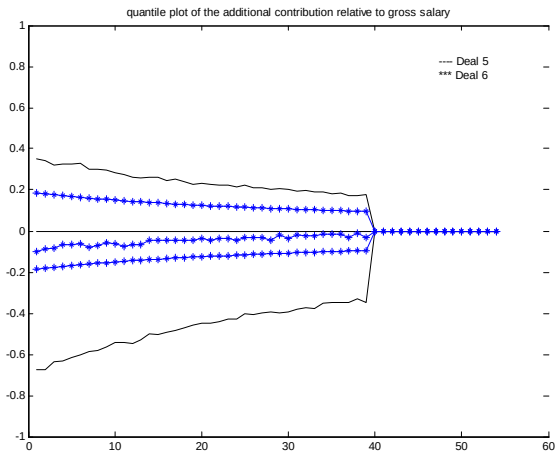

Figure 10: 20th 50th and 80th quantile of the actual benefit for Deal 3 and Deal 4 (50% Index-Linked Bonds and 50% Equity assumption)

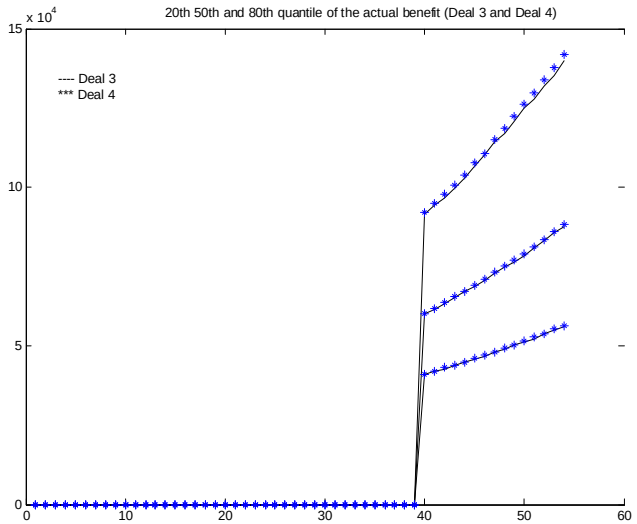

Figure 11: 20th 50th and 80th quantile of the actual benefit for Deal 5 and Deal 6 (50% Index-Linked Bonds and 50% Equity assumption)

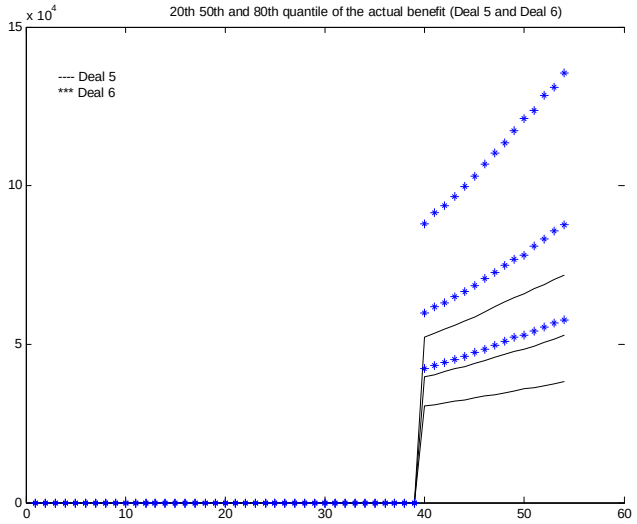

Figure 12: 20th 50th and 80th quantile of the actual benefit for Deal 3 and Deal 4, relative to state pension of 10000 euro (indexed) (50% Index-Linked Bonds and 50% Equity assumption)

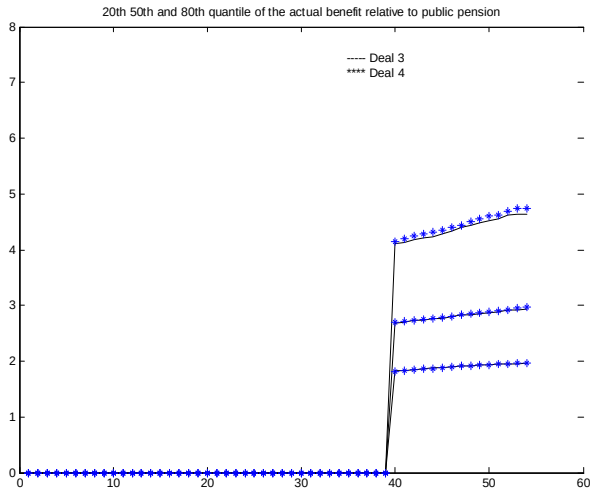

Figure 13: 20th 50th and 80th quantile of the actual benefit for Deal 5 and Deal 6, relative to state pension of 10000 euro (indexed) (50% Index-Linked Bonds and 50% Equity assumption)

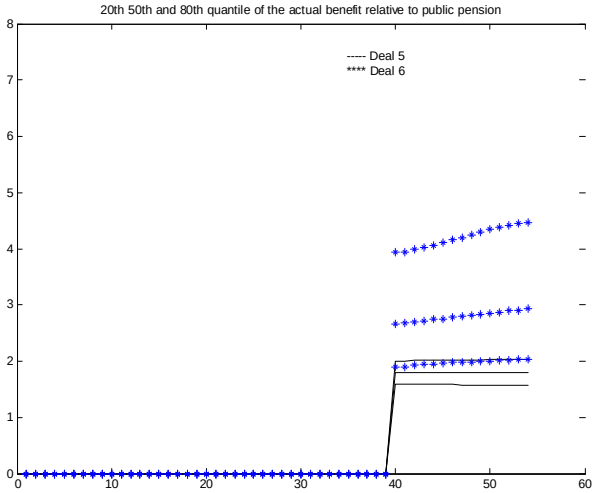

Figure 14: 20th 50th and 80th quantile of end-year Surplus for Deal 1 (thin line), Deal 2 (curve with dots) (50% Index-Linked Bonds and 50% Equity assumption)

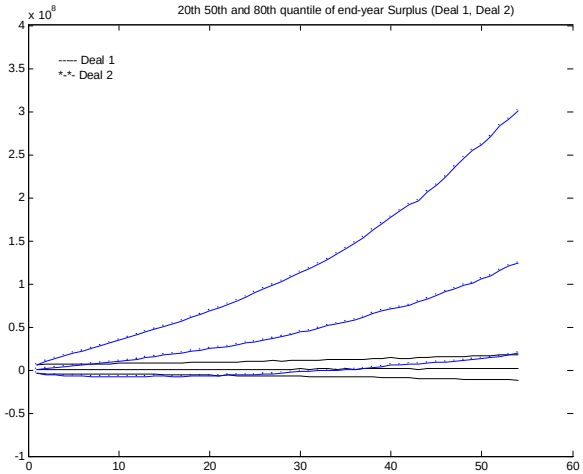

Figure 15: 20th 50th and 80th quantile of end-year Surplus for Deal 3 (thin line), Deal 4 (curve with dots) (50% Index-Linked Bonds and 50% Equity assumption)

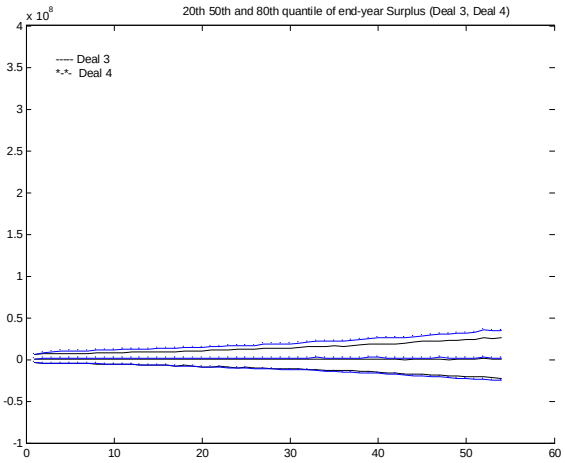

Figure 16: 20th 50th and 80th quantile of end-year Surplus for Deal 5 (thin line), Deal 6 (curve with dots) (50% Index-Linked Bonds and 50% Equity assumption)

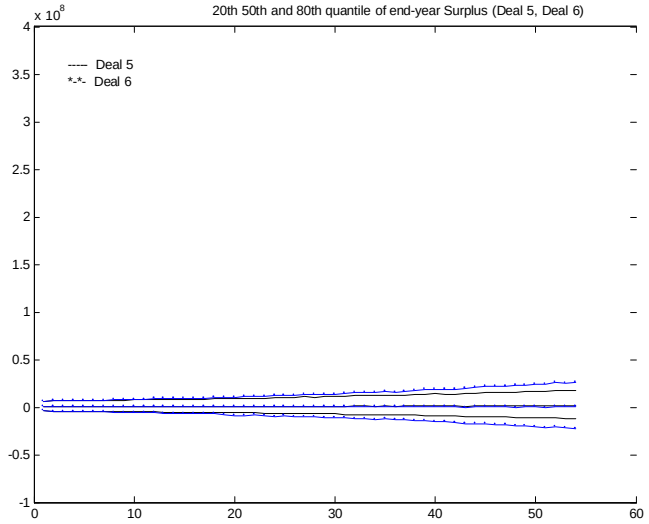

Figure 17: quantile of the additional indexation rates for Deal 3 and Deal 4 (50% Index-Linked Bonds and 50% Equity assumption)

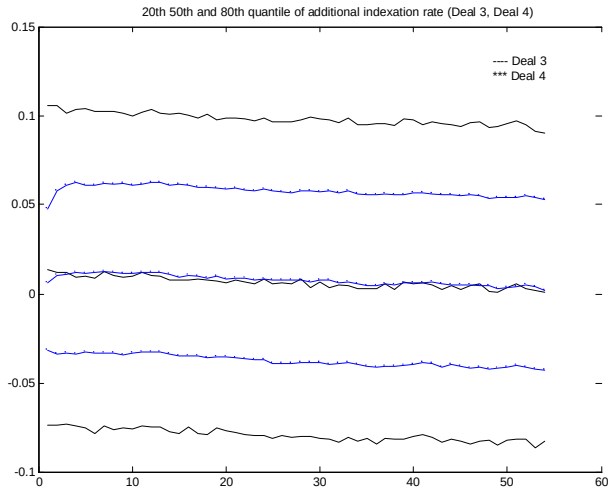

Figure 18: quantile of the additional indexation rates for Deal 5 and Deal 6 (50% Index-Linked Bonds and 50% Equity assumption)

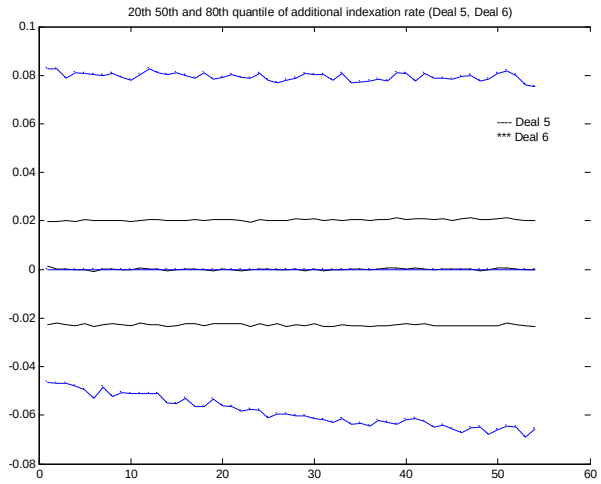

Figure 19: 20th, 50th and 80th quantile of the additional contribution for Deal 1 (thin curve) and Deal 2 (curve with stars) (50% Nominal Bonds and 50% Equity assumption)

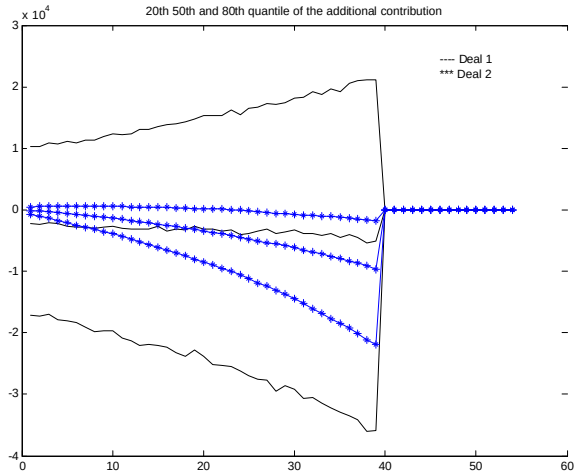

Figure 20: 20th, 50th and 80th quantile of the additional contribution for Deal 5 (thin curve) and Deal 6 (curve with stars) (50% Nominal Bonds and 50% Equity assumption)

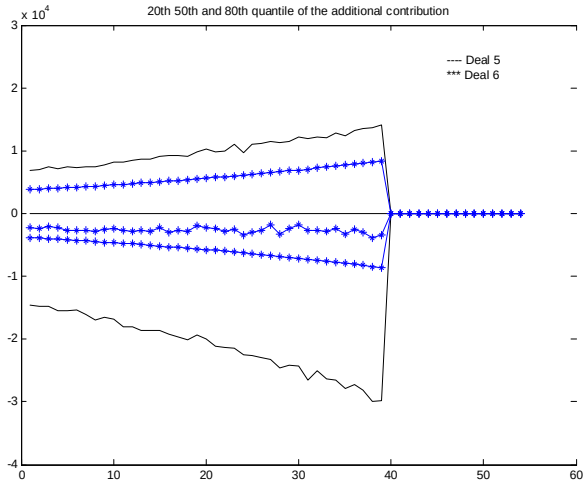

Figure 21: 20th 50th and 80th quantile of the actual benefit for Deal 3 and Deal 4 (50% Nominal Bonds and 50% Equity assumption)

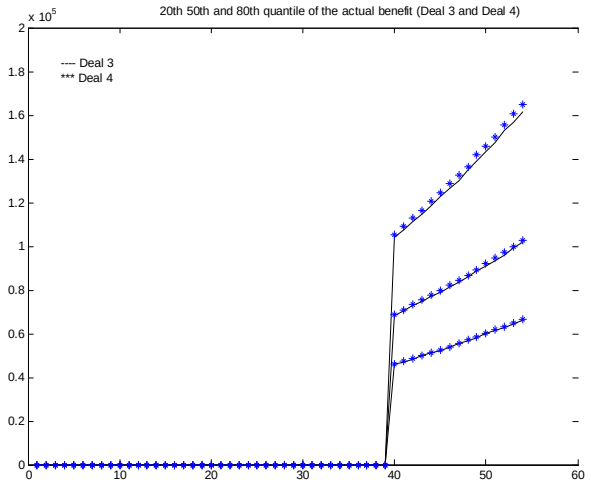

Figure 21: 20th 50th and 80th quantile of the actual benefit for Deal 5 and Deal 6 (50% Nominal Bonds and 50% Equity assumption)

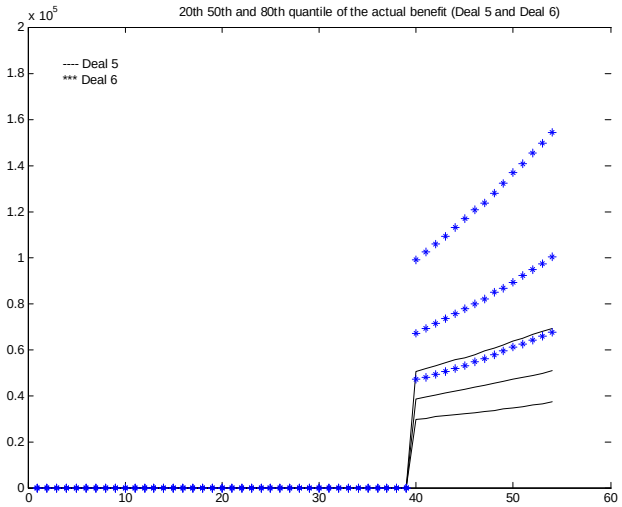

Figure 23: 20th 50th and 80th quantile of end-year Surplus for Deal 1 (thin line), Deal 2 (curve with dots) (50% Nominal Bonds and 50% Equity assumption)

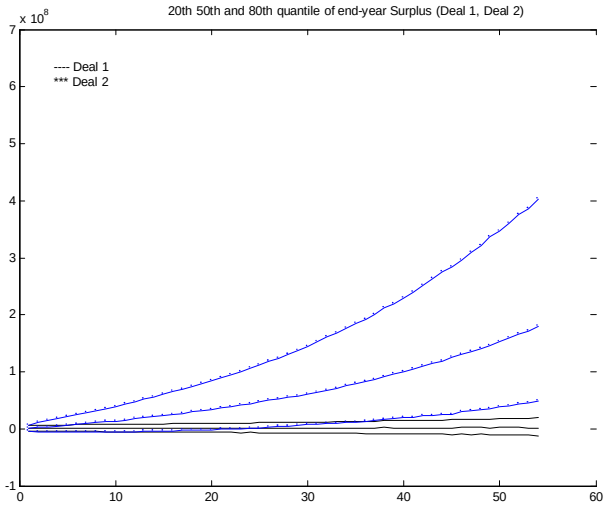

Figure 24: 20th 50th and 80th quantile of end-year Surplus for Deal 3 (thin line), Deal 4 (curve with dots) (50% Nominal Bonds and 50% Equity assumption)

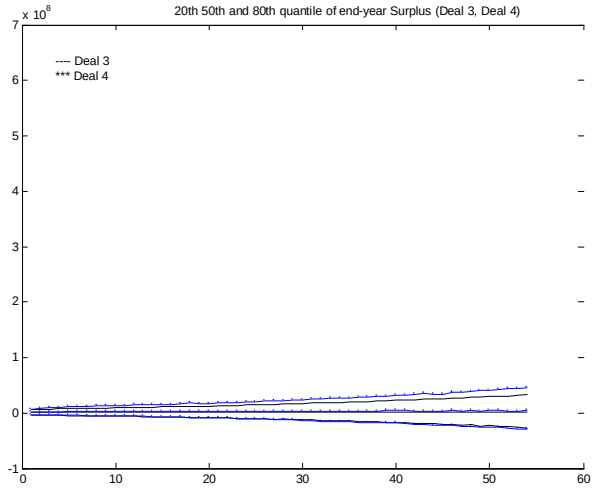

Figure 25: 20th 50th and 80th quantile of end-year Surplus for Deal 5 (thin line), Deal 6 (curve with dots) (50% Nominal Bonds and 50% Equity assumption)

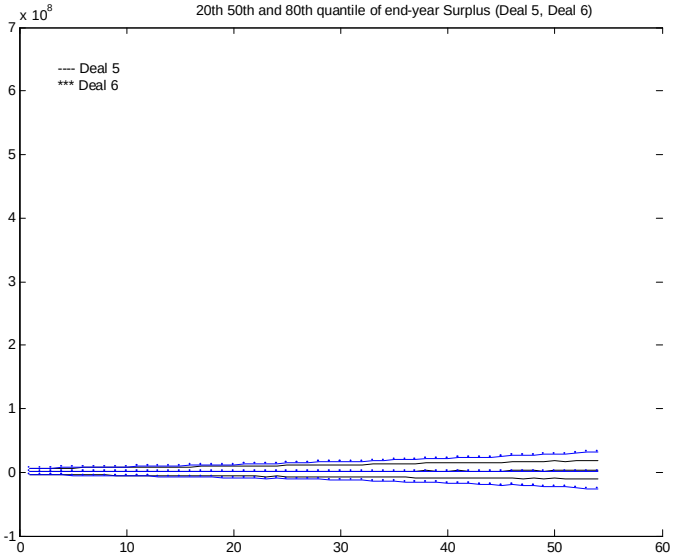

Figure 26: quantile of the additional indexation rates for Deal 3 and Deal 4 (50% Nominal Bonds and 50% Equity assumption)

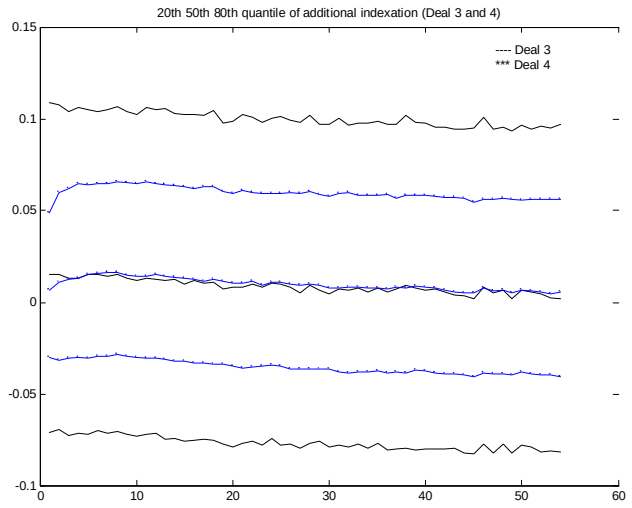

Figure 27: quantile of the additional indexation rates for Deal 5 and Deal 6 (50% Nominal Bonds and 50% Equity assumption)