When is a defined-benefit pension scheme too small for self-insurance?

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Motivation

Mortality risk components:

- Systematic risk - distribution of deaths.
- Idiosyncratic risk - random fluctuations.

Outline

Homogeneous pension scheme

- N members all age 40.
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- For example,

$$\mathbb{E}(L_N) = N\, v^{65-40}\, {}_{25}p_{40}\, \bar{a}_{65}.$$

Risk measure

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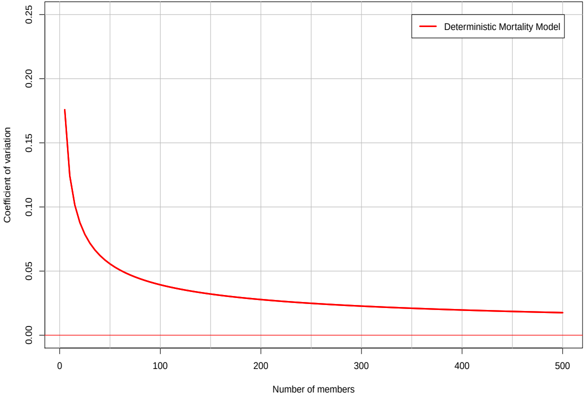
If $Y_1, Y_2, \ldots, Y_N$ are independent, then

$$\mathrm{VCo}(L_N) \quad = \quad \frac{\mathrm{sd}(L_N)}{\mathbb{E}(L_N)} \quad \to \quad 0 \quad \text{as } N \to \infty.$$

Numerical results: deterministic mortality model (PMA92C10) and $\delta = 4\%$ p.a.



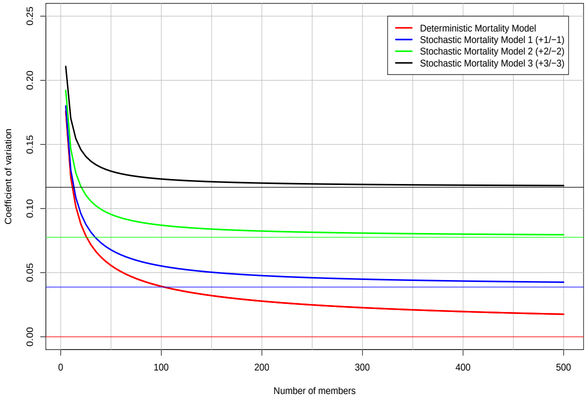
Stochastic mortality model

Stochastic mortality model r

$$\text{age rating} = \begin{cases} r & \text{with probability } 0.5, \\ -r & \text{with probability } 0.5. \end{cases}$$

based on PMA92C10.

Numerical results: $\delta = 4\%$ p.a.



Stochastic mortality model r

Here $Y_1, Y_2, \ldots, Y_N$ are not independent, and

$$\mathrm{VCo}_r(L_N) = \frac{\mathrm{sd}_r(L_N)}{\mathbb{E}_r(L_N)} \rightarrow \frac{\sqrt{\mathrm{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)} \quad \text{as } N \rightarrow \infty.$$

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Interpretation:

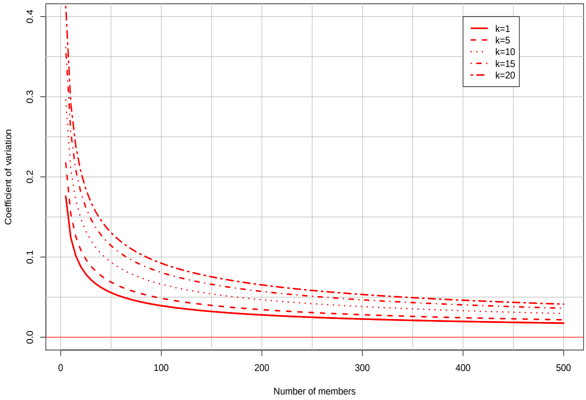
- Systematic risk measure:
$$\frac{\sqrt{\mathrm{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$
- Idiosyncratic risk measure:
$$\mathrm{VCo}_r(L_N) - \frac{\sqrt{\mathrm{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$

Pension scheme with executive section

As before except

- αN are executives.
- Executive benefit: £k p.a. from age 65.

Numerical results: deterministic mortality model (PMA92C10), $\delta = 4\%$ p.a. and $\alpha = 5\%$.



Pension scheme with executive section

- Setting

$$f(\alpha, k) = \frac{\alpha k^2 + 1 - \alpha}{(\alpha k + 1 - \alpha)^2},$$

we find

$$\begin{aligned} &\mathrm{VCo}_r(L_N) \\ =&\frac{1}{\mathbb{E}_r(Y_1)} \\ &\cdot \left(\frac{1}{N} f(\alpha, k) \left(\mathrm{Var}_r(Y_1) - \mathrm{Cov}_r(Y_1, Y_2) \right) + \mathrm{Cov}_r(Y_1, Y_2) \right)^{1/2}. \end{aligned}$$

Risk capital allocation

$$\boxed{\text{Risk capital} := \mathrm{sd}_r(L_N)}$$

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Find amounts $\pi_1, \pi_2, \ldots, \pi_N$ such that

$$\sum_{n=1}^{N} \pi_n = \mathrm{sd}_r(L_N).$$

Euler capital allocation principle

If X_n is P.V. r.v. of benefit due to member n then

$$\pi_n = \frac{\mathrm{Cov}_r(X_n, L_N)}{\mathrm{sd}_r(L_N)}$$

is the risk capital allocated to member n.

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If X_n is P.V. r.v. of benefit due to member n then

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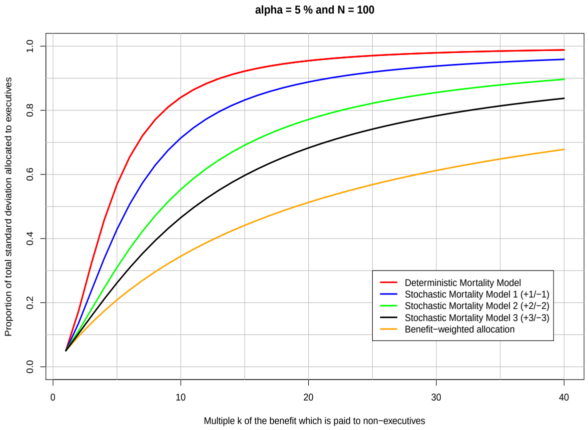
is the risk capital allocated to member n.

Consider

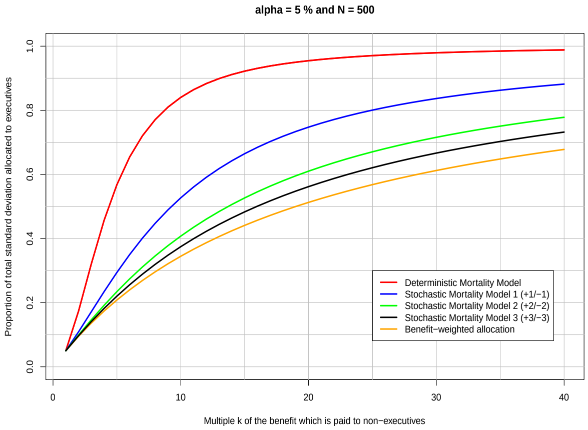
$$\frac{\sum_{\mathrm{execs}} \pi_n}{\mathrm{sd}_r(L_N)},$$

the proportion of risk capital allocated to executive section.

Numerical results: $\alpha = 5\%$, $N = 100$ and $\delta = 4\%$ p.a.



Numerical results: $\alpha = 5\%$, $N = 500$ and $\delta = 4\%$ p.a.



Euler capital allocation principle

We find

- systematic risk: execs contribute k times non-execs.
- idiosyncratic risk: execs contribute k times non-execs *plus*

$$k(k-1)\frac{\mathrm{Var}_r(Y_1) - \mathrm{Cov}_r(Y_1, Y_2)}{\mathrm{sd}_r(L_N)}.$$

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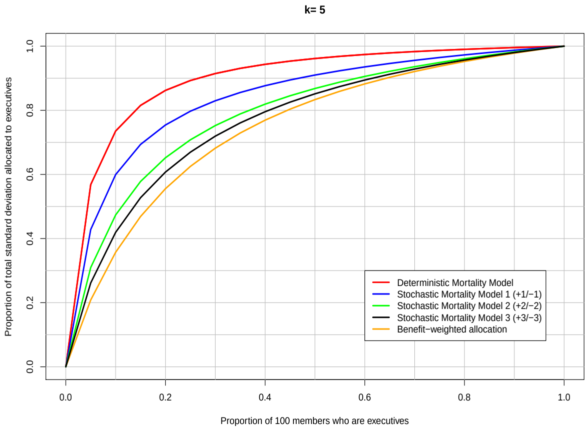

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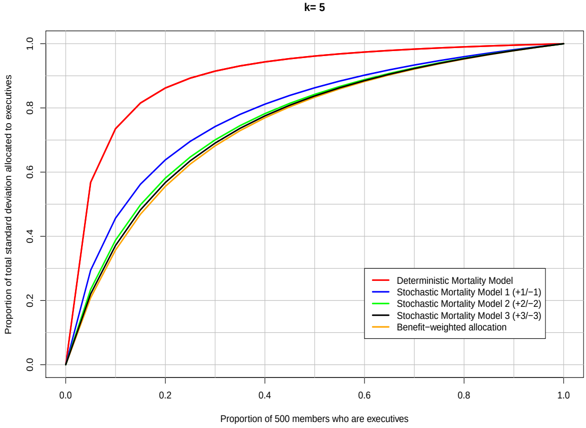

Future work

- More complex examples.
- Incorporate financial risks.
- Risk mitigation strategies.

Acknowledgements and references

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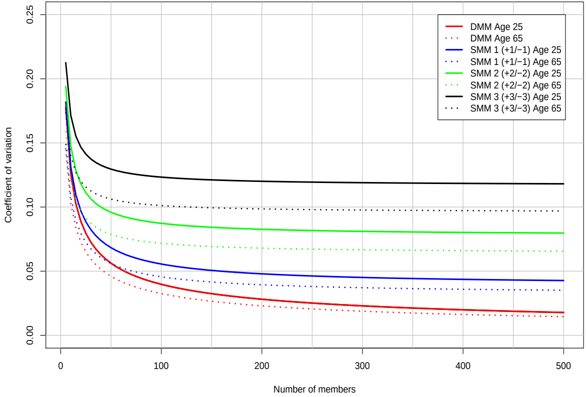
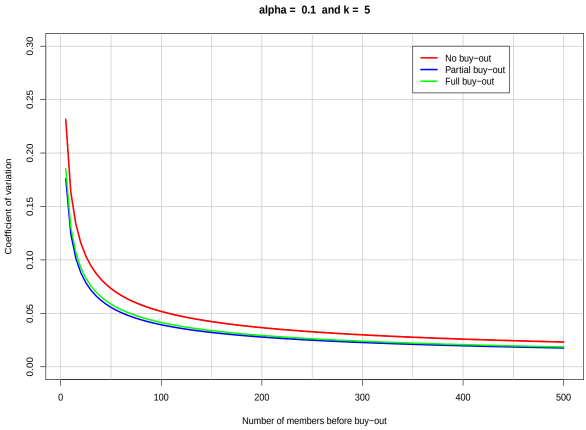
More details:

2011 C. Donnelly. *Quantifying mortality risk in a small defined-benefit pension schemes.* http://arxiv.org/abs/1107.1380.

Effect on VCo of change in age: $\delta = 4\%$ p.a.



Effect on VCo of executive buyout: $\delta = 4\%$ p.a.



Effect on standard deviation of executive buyout: $\delta = 4\%$ p.a.